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Basic Considerations in the Improvement of Elementary School Mathematics Programs*

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During the past few years the spotlights of attention on mathematics programs have increased in number and intensity and have broadened in focus. The glare of the limelights now covers the elementary school as well as the secondary school. It will continue to do so with increasing brightness for some time to come.

All this illumination is well intended. It is directed toward an over-all improvement of mathematics programs. Its central focal point is the mathematical content of the curriculum—its nature, scope, sequence, and the like; but its wider focus includes instructional methods, materials, activities, and experiences.

If these good intentions are to be realized most effectively in the elementary school, it is imperative that we give appropriate attention to a variety of things. Among those that need concern us are four which have been singled out for special consideration here.

1. Instructional Objectives

In the first place, our efforts to improve the elementary school mathematics program must take adequate cognizance of *all* instructional objectives. Some relate to the mathematics program alone; others, to the mathematics program as a part of the total elementary school program. In either event, each instructional objective must be kept in proper perspective in relation to the others.

In his discussion of "Arithmetic in 1970," Dean Brownell stated:

> Groups of mathematicians at the University of Illinois, the University of Maryland, and Yale University, together with commissions and committees of national mathematical organizations, originally motivated by the desire to improve secondary mathematics, are now turning their attention to arithmetic. It is too early to predict the fruitfulness of their explorations concerning the place of such mathematical ideas as set, symbolism, and verification in arithmetic; but they are certain to influence the arithmetic program of the higher grades and will probably affect to some extent that in the lower grades as well. As a matter of fact, their influence *could be too great* if their activities should produce a sterile arithmetic. By this term, I mean arithmetic as the pure science of number, taught with little regard for the ability of children to learn it or for the social purposes to be served. This danger, however, is unlikely to become a reality, for forces are available within the schools to counteract it. (1: 43–44)

Professor Buswell, in his discussion of "The Content and Organization of Arithmetic," stated:

> The main contention of the current critics of the schools is that arithmetic is failing to provide the solid basis of competence that is needed for high school and college programs in mathematics and

* Based on a paper presented at the 38th Annual Meeting of the National Council of Teachers of Mathematics at Buffalo, New York, on April 21, 1960.

science. We should give serious consideration to what the critics are saying. However, arithmetic is not taught for the sole purpose of training scientists. It must be organized so that all segments of the school population will be served in the ways that best fit individual needs . . . . Non-scientists also need arithmetic, and, still further, arithmetic is only part of the total enterprise of education. Schools are concerned with the good life to which arithmetic is only one of the contributors. While arithmetic teachers work to improve their own subjects, they must also maintain a sense of balance in respect to the total program of education. (2: 82)

The points made above are quite clear and need not be labored. Let us do all that we reasonably can to improve the elementary school mathematics program along those lines where improvement may be needed. Let us make sure that we do not end up with a doubtful or even a poor bargain, gaining some things at the expense of others of acknowledged importance. Our safeguard is to be found in the planning and implementation of improvement efforts in full cognizance of *all* relevant instructional objectives.

2. Present Status

In the second place, our efforts to improve the elementary school mathematics program should be based on a realistic and valid appraisal of present status. The steps to be taken toward our ultimate goals will depend a great deal upon where we are now in relation to where we eventually want to be.

We all recognize that at present the elementary school mathematics program leaves some things to be desired. We would be in a dangerous state of complacency if we felt otherwise! However, we may not always agree among ourselves regarding how good or how bad various phases of the program actually are at the present time.

Some persons take an extremely dim view of the current situation. Their appraisal of present status is epitomized well in the following statement made by Professor Van Engen:

In spirit, the present program is a complete stranger to mathematics; in content, it lacks modernity; in its exclusive attention to computational aspects of the subject, arithmetic as practiced in the schools is not mathematics. (3: 72)

Other persons take a slightly brighter view of the current situation. In fact, there are some who are much more encouraged than discouraged by the general direction in which we have been moving in elementary school mathematics during the past twenty to twenty-five years, and who are not wholly displeased with the present status of things. There is good reason why this less pessimistic feeling is a valid one.

Go back in your thinking to the classic first chapter of the Tenth Yearbook of the National Council of Teachers of Mathematics. There Brownell's comprehensive discussion of "Psychological Considerations in the Learning and the Teaching of Arithmetic" gave us the first well-developed statement of what was termed the "meaning theory" of arithmetic instruction. There, in 1935, we read these things, among others:

Arithmetic is best viewed as a system of quantitative thinking. . . . If one is to be successful in quantitative thinking one needs a fund of [mathematical] meanings, not a myriad of "automatic responses." If one is to adjust economically and satisfactorily to quantitative situations, one must be equipped to understand these situations and to react to them rationally. Specific responses to equally specific stimuli will not serve one's ends. (4: 10)

The "meaning" theory conceives of arithmetic as a closely knit system of understandable ideas, principles, and processes. According to this theory, the test of learning is not mere mechanical facility in "figuring." The true test is an intelligent grasp upon number relations and the ability to deal with arithmetical situations with proper comprehension of their mathematical as well as their practical significance. (4: 19)

According to the "meaning theory" the ultimate purpose of arithmetic instruction is the development of the ability to *think* in quantitative situations. The word "think" is used advisedly: the ability merely to perform certain operations mechanically and automatically is not enough. Children must be able to analyze real or described quantitative situations, to isolate and to treat adequately the arithmetical elements therein, and to make whatever adjustments are required by their solutions. (4: 28)

The basic tenet in the proposed instructional reorganization is to make arithmetic less a challenge to the pupil's memory and more a challenge to his intelligence. (4: 31)

These quotations reflect a point of view toward elementary school mathematics that twenty-five years ago was quite in line with many things being stressed today as we look

to the future. Actually, these quotations are wholly in the spirit of the recently published Yearbook of the National Council of Teachers of Mathematics, entitled *Instruction in Arithmetic* (5). During the past twenty-five years we *have* made progress—in some cases significant progress—toward refining and implementing that point of view. In fact, during much of this time the elementary school mathematics program was moving in a desirable direction more extensively and more rapidly than were the secondary school and college mathematics programs!

Do not misunderstand. No attempt is being made to "whitewash" the present elementary school mathematics program, implying that all is well. The need for improvement is recognized clearly, as is the fact that present practice frequently still lags behind theory. However, an attempt is being made to emphasize that efforts to improve the elementary school mathematics program must be based on a valid appraisal of present status; that a careful consideration of "things as they are" will reveal strengths as well as weaknesses; and that it would be a grave error to play up the weaknesses and disregard the strengths.

Professor Phillip Jones has voiced a caution regarding attempts to improve the secondary school mathematics program that, if heeded, can save us much possible grief in our growing desire to improve the elementary school mathematics program:

> . . . as I read and listen to the expositions of the denunciators and innovators I feel, happily, that with thought, discussion, and experimental teaching we are progressing toward a substantially improved mathematics program which will, however, be less changed and less radically modified than some of the loudest of the early outcries seemed to imply. . . . I think someone should speak up for "old" mathematics . . . and for conscientious teachers whose valuable experience in teaching "non-modern mathematics" should not be lost. (6: 66)

Our most effective improvement efforts will be those that take fullest advantage of the significant progress that has been made in a highly desirable direction and of the many good things currently being done as part of the elementary school mathematics program.

3. Means to Ends

In the third place, our efforts to improve the elementary school mathematics program must take into account the significant fact that often there is more than one acceptable means to the same desired end. We must distinguish carefully between *a* way to accomplish something, and *the* way (if there ever is such a thing). It is recognized, of course, that there are occasions when one way may be preferable to another; but in most instances this is a highly relative rather than an absolute matter. Concern here is for the fact that too often we are being told, in effect, that one particular way is the *right* way, and that virtually all other ways are wrong, when such is not the case at all.

This unfortunate point of view is encountered in connection with the nature and organization of the mathematical content of the curriculum, the mathematical procedures studied, the methods and materials of instruction used, and the like. We need become very much concerned when we are told, in essence:

a) that a particular set of instructional materials is the *only* really mathematically correct set of representative materials available for our use, or that a particular instructional program is the *only* really mathematically correct program in existence;

b) that one particular method of instruction is the *only* psychologically correct method to be used;

c) that one particular way of organizing a body of mathematical content is the *only* correct way of doing so, or that a particular sequence of development is the *only* mathematically correct one; or

d) that one particular mathematical procedure or algorithm is the *only* mathematically correct one to use.

Of course, there are things we have done on occasion in the past and are doing at present in some instances that essentially are mathematically *in*correct. We need do all that we can to remedy such situations. However, in so doing, we should not overlook the fact that often there is more than one way

in which mathematical correctness can be achieved.

The hope for an improved elementary school mathematics program is not to be realized through a search for *the* instructional program, in the sense of there being but *one* program that is mathematically and psychologically "correct." Such a search would be fruitless, for reasons that should be obvious.

In the days ahead we have an unprecedented opportunity to tackle the problem of improving the elementary school mathematics program on many more fronts than ever has been possible in the past. Our so-called experimental programs, and other improvement efforts, can do much to explore *various* possible ways in which this improvement can be effected. We should welcome work that is being done in the true spirit of exploration and experimentation. But we should look with justified suspicion on any work that purports to have *the* answer to all facets of an improved elementary school mathematics program. As we search for those things that will improve the mathematics program at the elementary school level, let us seek to find *various good* means to the common end we all desire.

4. Teacher Education and School Organization

Finally, in the fourth place, we must examine more carefully than ever before our prevailing form of elementary school organization in light of the need for improved classroom instruction and teacher education.

Many persons recognize the problem of teacher education to be the most critical problem we face in our efforts to improve the elementary school mathematics program. The crux of the problem is *mathematical* in nature. Ruddell, Dutton, and Reckzeh stated the matter simply and concisely in the Twenty-fifth Yearbook of the National Council of Teachers of Mathematics:

> The careful preparation of prospective teachers in *mathematics subject matter* is a prerequisite to an improved program in arithmetic in elementary schools. [Italics mine.] (5: 297)

In dealing with this critical problem of teacher education, we must be careful to distinguish between the *immediate* problem and the *long-range* problem. For purposes of our present discussion, attention will be focused only on the latter: the long-range problem.

Pause and reflect for a moment on the mathematical background desired in an elementary school teacher. Then project your reflection to content background desired in an elementary school teacher in other subject-matter fields: social studies, science, the language arts, and the like. There is good reason to question the possibility of training an elementary school teacher, within a finite period of time, so that he will have the desired content background in each of these subject-matter fields—to say nothing about other aspects of his total program of professional preparation.

For too long we have tried to do the impossible; i.e., in training teachers for the elementary school we have tried to expect them to provide high-level instruction in virtually all subject-matter fields. As a consequence we have developed entirely too many elementary school teachers who are simply "Jacks-of-all-trades and masters of none."

There would seem to be distinct advantage in teacher-education programs which develop in each trainee a particular field of subject-matter competence; i.e., a field of subject-matter specialization. For some persons this would be the field of mathematics. Fortunately, some teacher-education programs already have been designed to accomplish this purpose. In the future, however, many more of our teacher-education programs must be similarly redesigned.

Another important step must be taken at the same time. Not only must we redesign teacher-education programs so as to provide teachers with special training and competence in mathematics, but we also must restructure many of our elementary school organizational patterns so as to use most effectively each teacher's subject-matter specialty. The "self-contained classroom" as it exists in many schools today is more of a

hindrance than a help in relation to the improvement of elementary school mathematics programs.

We need much experimentation with a variety of patterns of elementary school organization, each designed to utilize to best advantage teachers who have been trained in part as subject-matter specialists. Some schools already are structured in ways that permit realization of this goal to one degree or another. Other schools are experimenting with various departures from the "self-contained classroom" organization. Some of these explorations are part of large-scale investigations, such as the AAAS "Study on the Use of Special Teachers of Science and Mathematics in Grades 5 and 6" (7). In any event, we need much more experimentation with patterns of elementary school organization that will permit the optimum use of teachers specially trained in the field of mathematics. Ultimate improvement in elementary school mathematics programs depends in large measure upon desirable changes in both teacher education and school organization.

In Summary

The foregoing discussion has focused attention on four basic considerations in the improvement of elementary school mathematics programs:

1. the need for keeping improvement efforts in proper perspective in relation to *all* relevant instructional objectives;

2. the need for basing improvement efforts on a valid appraisal of present status, with full recognition of strengths that already exist in current programs;

3. the need for recognizing and finding various acceptable and effective means for reaching a common end or goal; and

4. the need for long-range reconsideration of teacher education and school organization with a view toward optimum use of specially trained teachers in the field of mathematics.

Other considerations must receive our attention, of course. However, the four basic considerations discussed here are particularly crucial ones. The extent to which we make significant progress toward improved elementary school mathematics programs will be in direct proportion to the attention we give the four considerations that formed the basis for this discussion.

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Editor's Note. Professor Weaver has pointed out several things that are important to all those who are working long and hard in the elementary field.

The first is a timely warning against succumbing to a "band-wagon psychology" and risking present gains by adopting the new merely for the sake of newness.

The observation that the elementary mathematics program has been moving in a desirable direction for some time, despite the lack of headline space and extensive support by public funds, must be a comfort to many of us.

His third point, that there is usually more than one way to do almost anything, supports his fourth. Only when arithmetic teachers are well trained, and only when the teaching of arithmetic is in competent hands, will we feel reasonably certain that the right material, the right method, and the right algorithm or procedure for a particular set of circumstances is being presented.

The Point of View of the Twenty-fifth Yearbook*

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Instruction in Arithmetic, the Twenty-fifth Yearbook of the National Council of Teachers of Mathematics, is the third yearbook published by the Council which deals with arithmetic. The present publication presents the point of view that was introduced in previous yearbooks in this area. To a large extent, therefore, this book represents a progress report dealing with the acceptance of ideas that were initiated in earlier yearbooks. For this reason it seems advisable to review the philosophy of learning as applied to arithmetic in former yearbooks in this field.

The Tenth Yearbook, published in 1935, was the first publication dealing with arithmetic which was issued by the Council. This book made a marked contribution to the literature in this field because of the theory of learning it described. Wheeler proposed that learning does not consist so much in forming bonds as it does in establishing patterns of thinking. He suggested that more emphasis should be placed on the relation of parts to the whole and less on the formation of a hierarchy of skills or separate facts. The chapter by Wheeler was unique in challenging the atomistic theory of learning as outlined predominantly in the Twenty-ninth Yearbook of the National Society for the Study of Education, which had been published five years earlier.

Although the Tenth Yearbook suggested a structure or Gestalt as essential in understanding arithmetic, the book did not give great emphasis to this view. On the other hand, this publication showed how the informational phase of arithmetic should merit consideration. Emphasis on informational arithmetic represented a very effective means of enabling the teacher to understand that drill in computation should not consume such a large part of the program as characterized the teaching of the subject at that time.

Six years later the Council published the Sixteenth Yearbook dealing with the same subject. This yearbook reiterated the view of learning as based on discovering patterns or structure in number. McConnell showed that organization and structure are vital elements in learning arithmetic. He showed that learnings need to be made permanent; hence drill is an essential part of a program for learning arithmetic. In Wheeler's presentation of the subject, drill received little or no consideration.

One of the most significant contributions of the Sixteenth Yearbook was the formulation of a plan to implement a program which would emphasize meaning and understanding. Irene Sauble showed how supplementary materials should be considered essential learning aids in helping the pupil to discover the structure of the number system and relationships among numbers. The emphasis on developing meanings supplemented by drill plus the implementation of a program stressing these facts represented two major contributions of the Sixteenth Yearbook.

The third feature of this yearbook was its delineation of the social and mathematical phases of arithmetic. The social phase, or its significance, refers to the application of

* Presented at the 38th Annual Meeting of the National Council of Teachers of Mathematics, Buffalo, New York, April 21, 1960. Mr. Grossnickle is editor of *Instruction in Arithmetic*, the Twenty-fifth Yearbook of the National Council of Teachers of Mathematics.

arithmetic in our daily affairs. The mathematical phase refers to the relationships among numbers or quantities. Unfortunately, the demarcation between these two phases often led to a dichotomy that was not conducive to effective learning in this area. Some teachers gave disproportionate emphasis to the social phase. Similarly, other teachers stressed the mathematical phase. The demarcation between the two phases militated against making arithmetic a unified subject in dealing with quantities.

The Fiftieth Yearbook of the National Society for the Study of Education, published in 1951, presented the view that learning results from the discovery of a pattern or a Gestalt of a structured whole. "Structure," used here, conveys about the same meaning as it does in its common usage. The structure of a building gives shape and rigidity to the edifice. In the same way the structure of arithmetic provides the framework for the operation of a system of numeration. The field psychologist looks upon the structured whole of arithmetic rather than upon the composite of unrelated elements or parts. The writings of Wheeler, McConnell, and Buswell are representative of the views of the field psychologist.

Your Yearbook Committee endorsed this philosophy of learning. Therefore, the major theme of this yearbook is the problem of implementing a program based on this view. Although the Fiftieth Yearbook introduced the view described, there have been many significant changes in elementary mathematical education since the publication of that book. Some of these changes had to be recognized in the development of the present yearbook.

Instruction in Arithmetic shows that there are three important phases in an effective program in arithmetic. First, the subject has a cultural value; it is structured; if properly taught, it leads to efficient ways of quantitative thinking; and it is basic in the further study of mathematics. Second, a program in arithmetic should make provision for individual differences among learners, have adequate records for effective counseling and guidance, recognize the part mental hygiene plays in a learning situation, supply different instructional materials, and offer adequate instruction in reading and interpreting quantitative statements. And third, an effective program in arithmetic should provide teachers with a background in mathematics adequate to give successful instruction.

There always is a lag between accepted theory and practice in any program of instruction in our schools. One of the major objectives of the yearbook is to help supervisors and administrators to develop an in-service program of instruction for teachers so as to meet the demands of a modern program in arithmetic. These teachers, as well as prospective teachers in colleges of education, should develop an understanding of the structure of arithmetic, the value of proper mental hygiene in the classroom, the function of guidance and counseling, how to use and prepare materials, and how to provide for the span of ability within groups. These are some of the topics presented in this present publication.

The chapters of the yearbook may be grouped into four parts. Part 1 deals with the cultural value of arithmetic, the structure of arithmetic, and learnings peculiar to this subject. Part 2 deals with the learner and factors which affect the learning of the subject. Part 3 deals with the preparation of teachers. Part 4 deals with research investigations made in arithmetic since the publication of the Sixteenth Yearbook approximately twenty years ago.

Two topics usually given in a yearbook are missing from the Twenty-fifth Yearbook. One of these is a treatment of the curriculum and the other a treatment of a program for testing and evaluation. These omissions were made because the Council plans to give a more detailed presentation of each of these topics than could have been given in this yearbook. The Twenty-sixth Yearbook will deal with testing and evaluation. A committee under the chairmanship of Fred Weaver has been at work for the past few years preparing a report dealing with the curriculum in arithmetic.

Instruction in Arithmetic devotes one chapter to the curriculum for grades 1 and 2. This area is the most unexplored part of the curriculum in arithmetic. For this reason the Yearbook Committee considered it advisable to give a report describing the work to be covered in this subject in these two grades. Spitzer proposed two significant principles to be recognized in planning the work. First, the principle of discovery should be applied in these grades as it is assumed to be applied in later grades. Second, the major portion of the time assigned to teaching number in these grades should be given to the development of quantitative concepts. Although the development of number concepts is essential, the pupil should also have a mastery of the basic facts of addition and subtraction by the end of the second grade. There should be no starvation diet in arithmetic in these grades, as is true of the curriculums followed in many school centers. A program calling for enriched development of number concepts plus a mastery of the basic facts in addition and subtraction represents a conservative amount of number work for grades 1 and 2. This is especially true since there is at present a noticeable clamor in the press for an increase in instruction in mathematics and science in the elementary school.

For purposes of this discussion, the topics in the Twenty-fifth Yearbook are divided into four parts. This division is given for discussion purposes only; there is no such demarcation in the publication. Part 1 deals with the cultural value of arithmetic, the structure of the subject, and learnings peculiar to arithmetic.

At a meeting of the New Jersey Mathematics Association in May, 1959, President Newsom of New York University gave an intriguing definition of arithmetic. He defined arithmetic as "the mathematics of the layman." I would supplement this definition by stating that "arithmetic is the alphabet of the mathematics," or that "arithmetic is the embryo of the mathematics." The use of these two different definitions makes it possible to evaluate the attempt made by the yearbook to structure arithmetic. Arithmetic is the mathematics which the man in the street uses in finding the answers to certain quantitative situations met in his daily affairs. He is very little concerned about the operation of the distributive law when he multiplies to find how much his earnings will be in eight hours at $2.75 per hour. He is concerned with multiplication only as a tool in order to find the answer to a vital problem. According to the second definition, an understanding of the structure of arithmetic is basic for further study in mathematics. The future scientist should be greatly concerned with the laws or principles governing arithmetic.

Instruction in Arithmetic contains three chapters written, respectively, by Van Engen and Gibb, by Clark, and by Swain. Each writer describes in whole or in part the structure of arithmetic in terms of the five basic laws which govern the operations within the framework of the number system. These are listed as the *commutative*, *associative*, and *distributive* laws, and the two identities, or the *law of one* and the *law of zero*.

There is no uniformity of phraseology used to designate these laws. Van Engen and Gibb use the term *principles*, Clark uses the term *generalizations*, and Swain uses the term *laws*. Some textbooks in arithmetic use the term *mathematical ideas*. The manner of designating these concepts in the elementary school is of no significance.

Clark lists a total of thirteen generalizations which are basic in the study of arithmetic. A generalization or a principle is a consequence of one of the five fundamental laws previously mentioned. Thus, an important principle dealing with common fractions states that both terms of a fraction may be multiplied or divided by the same nonzero number without changing the value of the fraction. This generalization or principle is a derivative or a consequence of the identity or the *law of one*. In the same way the other important ideas or principles are consequences of the five basic laws.

There are two factors to be considered in dealing with these generalizations. First,

there is no fixed number, such as the thirteen generalizations described by Clark. It is easy to add to or delete one or more from the list which he gives. Second, the manner in which the pupil learns the principle is as important as the principle itself. Clark has shown a pattern which the teacher may use to discover or to make the generalization. The discovery of the principle is the important part of the learning process. These principles or generalizations are not a body of subject matter to be mastered. Instead, they constitute the rules which govern the procedures to be followed in the various basic operations.

A difficult question to be answered is whether arithmetic should be considered as the mathematics of the layman *and* the alphabet of the mathematics, or whether it is to be considered as either one *or* the other of these interpretations. Should arithmetic be structured for all pupils or should this phase of the subject be limited to the more able pupils? Personally, I feel that the slow learners profit little from attempting to learn much concerning the structure of arithmetic. On the other hand, the more able pupils most certainly should be able to discover the basic rules or principles which govern the arithmetical operations or processes.

In most courses of study in arithmetic certain goals of achievement are established for different grade levels. These goals should include an understanding of certain mathematical ideas or principles. Thus, at the end of the third grade the pupil should understand that the order of adding or multiplying two numbers does not affect the answer. The pupil should use the term "order," which is descriptive of the process, instead of using the term, "commutative law." He also should understand that changing the order of two unequal numbers in subtraction or division changes the answer. It is almost certain that most teachers have the pupil discover for himself these elementary principles in dealing with the basic facts in addition and multiplication. However, only a limited number of teachers realize that an understanding of these basic principles constitutes a vital element in structuring arithmetic for more advanced work in elementary mathematics.

It has been pointed out in a number of instances that such concepts as laws, principles, generalizations, and mathematical ideas are used interchangeably in different publications. This is inevitable because there is no standard terminology by which to express these concepts. The Yearbook Committee was conscious of this deficiency and appointed a committee composed of Boyer, Brumfeld, and Higgins to define certain concepts used in arithmetic. This committee of three made a commendable beginning in dealing with this problem.

The chapter dealing with definitions has two parts. Part 1 is subdivided to contain definitions of two types of concepts: those that are comparatively well known, and those that are widely used in measurement, but often are used erroneously. The use of the terms "correct" and "accurate" typifies the concepts in the latter group. Part 2 defines concepts dealing with modern mathematics. This list includes certain concepts associated with sets and set theory.

It is to be expected that many teachers will not agree with all the definitions given. Teachers also will soon discover that all mathematical terms used in arithmetic are not included in the chapter. It is the hope of the planning committee of the yearbook that the list of concepts defined in this publication may serve as a nucleus for the listing of additional definitions of other basic concepts used in this subject. The task of revising and adding to the list of terms given should constitute a future obligation of the Council.

The second classification of topics given in the yearbook deals with the learner and the factors affecting learning. The coauthors of the five chapters included in this section are Jones and Pingry, Sauble and Thiele, Dickey and Taylor, Spencer and Russell, and Hardgrove and Sueltz.

The major problem in any area of instruction is to provide optimum learning conditions so that each pupil will understand and succeed at his own level of operation. Jones

and Pingry show how to utilize both administrative procedures and differentiation of the curriculum to provide for the range of abilities in a class. The authors not only discuss such controversial problems as the practice of acceleration and ability grouping, but also give their views on these issues.

Sauble and Thiele show the kinds of records needed for proper guidance of the pupil in arithmetic. These writers recommend that an effective program in guidance begin when the pupil enters school. It should not be deferred until the junior or senior high school. The writers point out the special need for guidance of the gifted and offer many practical suggestions for dealing with this group.

The mental hygiene of the classroom is an important factor in learning. Since many pupils form mental blocks unfavorable to learning arithmetic, a chapter dealing with the formation of the proper attitude toward arithmetic seems essential in a yearbook on this subject. Dickey and Taylor show how intangibles are vital factors in learning the subject. The writers formulate certain principles of good mental hygiene to be followed in teaching.

Reading in arithmetic presents a different problem from reading for pleasure. Reading difficulty has always been one of the major obstacles to progress in problem solving. Spencer differentiates between primary reading and secondary reading. He defines primary reading as that form of reading which occurs in the written problems in a text. Secondary reading involves directions for exercises, tables, graphs, and similar types of activities. Each form of reading requires the development of special skills. Russell gives a summary of research related to the question of reading in problem solving, and enumerates a list of helpful aids which the teacher may use to develop the pupil's *power* in reading. A pupil who develops power in reading usually is able to understand verbal statements which express quantitative situations in problems.

The use of various kinds of supplementary aids represents a great change in the teaching of the subject within the past ten or twenty years. At present there are so many manipulative, visual, and audio-visual materials available that the teacher must exercise great care in the selection and use of these materials. Hardgrove and Sueltz propose criteria for the selection and use of these supplementary aids. The list of materials these authors give should assist a teacher in deciding upon the kind of supplementary aids to be used in the arithmetic classroom.

The third classification of topics of the yearbook deals with the mathematical preparation of teachers of arithmetic. Ruddell, Dutton, and Reckzeh propose a list of topics to be included in a background course for teachers of arithmetic. These writers base their recommendations on the results obtained from a questionnaire study sent to a representative sampling of mathematics departments of state colleges for the preparation of teachers. This study was made in 1956. If the study had been made two or three years later, it is almost certain that the recommendations would include some topics which are now missing. The impact of modern mathematics had not been widely felt in 1956. A few years later set terminology became a common topic in discussions of the background preparation of teachers. The recommended background course makes no mention of modern mathematics.

The report dealing with the academic preparation of teachers of arithmetic shows that considerable progress has been made since the report given on this subject in the Fiftieth Yearbook. Although progress has been made in achieving higher standards of attainment in this field, the desired goal of the minimum preparation of teachers of arithmetic has not yet been reached. The current yearbook repeats the recommendation of the Fiftieth Yearbook. In each case the minimum preparation of teachers consists of six semester hours of background work in mathematics at the college level plus three semester hours in the teaching of the subject. The number of hours of background mathematics is not so important as

the nature of the course. The background course recommended by the writers mentioned gives emphasis to the kind of preparation in arithmetic that teachers of this subject should have.

The fourth and final classification of topics in the yearbook contains a resume of the investigations and significant articles dealing with arithmetic which have appeared since the publication of the Sixteenth Yearbook. Schaaf lists over 200 studies which he grouped under fourteen different classifications. There were many more articles and investigations published during that period than those included in the list; Schaaf was the judge of whether or not a publication should be included in this study. Schaaf is eminently qualified for a task of this kind, and students wishing to investigate a particular topic in arithmetic should find his chapter of great value.

By way of summary, *Instruction in Arithmetic* may be considered a progress report dealing with ideas that were introduced in earlier yearbooks pertaining to the structure and learning of arithmetic. Such concepts as meaning, understanding, and structure are more clearly defined in this publication than they were in similar previous publications.

The Twenty-fifth Yearbook treats three basic topics. They are: first, the nature of arithmetic; second, the learner and factors affecting him; and third, the preparation of the teacher. A program which gives adequate recognition to these three factors is certain to meet the standards of a modern program in arithmetic. Your Yearbook Committee hopes that *Instruction in Arithmetic* will help to implement a modern program in this subject.

Editor's Note. Professor Grossnickle's summary of the objectives and content of the Twenty-fifth Yearbook is understandably conservative. As editor he is to be congratulated on the scope and quality of the book. It is to be hoped that this summary will stimulate its use at all levels, from public school classroom teachers to college professors. It deserves to be widely distributed.

Announcement

The Arithmetic Teacher will henceforth be issued eight times per year during the months October through May. The Board of Directors voted this change at the annual meeting held in Dallas last April. Thus the two journals, The Mathematics Teacher and The Arithmetic Teacher, will have the same number of issues annually. In six years The Arithmetic Teacher has grown from zero to ten thousand circulation. The National Council is happy to increase the number of issues from six to eight in token of the fine reception accorded to The Arithmetic Teacher.

A Study of the Attitudes Toward Arithmetic of Students and Teachers in the Third, Fourth, and Sixth Grades

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IN THE LAST few years many changes have occurred in the teaching of arithmetic, with the result that today arithmetic has a place of much greater importance in the curriculum. However, even with these changes, much of the current literature about arithmetic in the elementary curriculum gives one the impression that arithmetic is still a much disliked subject. Statements such as these appear in periodicals: "It is only too certain that today's mathematically ill-prepared teachers, many of whom are ill-disposed toward the subject, are infecting too large a number of our boys and girls with an enduring fear and hatred of mathematics, which can rarely be overcome later on in high school,"[1] and "Most students who have a fear and dislike of mathematics met with some frustration in the elementary grades."[2]

In the *New York Times* this statement has appeared:

> Attitudes of frustration build up because of insufficient challenge or because of too difficult work in the elementary grades. The students of today's classroom represent widely different capacities and interests which cannot be satisfied through uniform content and method . . . . The future of many American scientists and mathematicians depends on how they feel about mathematics in the early grades.[3]

B. R. Buckingham says:

> One of my colleagues at Ohio State University used to dismiss arithmetic with the remark—often repeated—that the subject had come to a standstill, that there was little more to be learned about it, and that those who concerned themselves with it were dealing with trivialities. We knew all we needed to know, said he, about arithmetic, and all of any consequence that we were ever likely to want to know. I fancy too that my colleague, if he had spoken his full mind, would have said that arithmetic is a hard subject, an unlovely subject, and a subject altogether ungrateful, demanding the strength of the young and repaying with disappointment.[4]

Just what is the attitude taken by children in the elementary school today? Are teachers really infecting fear and hatred into a large number of our boys and girls? Is arithmetic still a much disliked subject with little practical application in our everyday life?

It was with these questions and others in mind that the writer undertook a study to attempt to discover within a limited field the present-day attitudes of children and teachers toward arithmetic.

The writer will feel justified in having made the study if an elementary teacher who reads this report asks herself, "Is this the attitude of the children in my arithmetic class?" "Is arithmetic meaningful to my group?" "Do 17% of the children in my room find arithmetic boring?"

What can the teacher do for the child who wishes he didn't have to be submitted to arithmetic? Is it possible through carefully planned lessons to change or at least modify a child's negative attitude?

[1] Marshall Stone, "Fundamental Issues in the Teaching of Elementary School Mathematics," THE ARITHMETIC TEACHER, VI (October, 1959), 177.

[2] Leon McDermott, "A Study of Factors That Cause Fear and Dislike of Mathematics," Dissertation Abstract 19, July, 1958, M.Ed., Michigan State University, p. 71.

[3] "Feel For Science Develops in Youth," *New York Times*, February 18, 1957.

[4] B. R. Buckingham, "Perspective in the Field of Arithmetic," THE ARITHMETIC TEACHER, II (February, 1955), 1.

Statement of the Problem

Most educators agree that attitude plays an important part in the learning process. Attitudes formed early in life quite often persist throughout life. Elementary schools, therefore, seem to have a great responsibility in helping to create favorable attitudes toward school subjects.

How are attitudes formed? Do children as early as third grade have rather firm attitudes toward arithmetic? Do teachers work to establish a wholesome frame of mind in children concerning the need for and importance of arithmetic? Does the teacher use many devices and employ different methods with different children, or is arithmetic still merely the drill subject that it once was in the curriculum?

The purpose of this project is to study the attitudes of children and teachers today, to note changes, if any, from third grade to fourth grade to sixth grade, to note trends in attitudes of both children and teacher, and to compare the attitudes of boys and girls toward arithmetic.

Nature of the Study

The study was begun by searching for an attitude scale for school subjects. There have been many scales devised and used successfully for elementary school subjects. Dutton's Attitude Scale appeared adequate, but careful study proved the need of revision for the purpose the writer had in mind.

Dr. H. E. Remmers of Purdue University's Division of Educational Reference has used many types of attitude scales and graciously supplied samples for study. Dr. N. L. Gage of the University of Illinois also supplied information which was helpful.

After careful study of these samples and after several trials with small groups of children and a sample of teachers, the Dutton Attitude Scale was revised for purposes of this study.

Limitations of the Study

The writer realizes that this research study has definite limitations.

1. No attempt has been made to compare achievement with attitude.
2. There has been no provision made for individual differences in children's abilities to read and interpret the attitude scale.
3. No attempt has been made to discover why some children take unfavorable attitudes toward arithmetic.
4. There has been no follow-up to note change in attitude from month to month or year to year.
5. The area studies have been limited to selected school districts in Pennsylvania and one in Ohio.

Procedure

The writer selected for this study children from grades three, four, and six. The study was pursued through the co-operation of the teachers and elementary supervisors from the following schools in Pennsylvania: Benjamin Franklin Joint School, Indiana County; the Penns Manor District, Indiana County; West Deer Fraser School in Allegheny County; Latrobe City Schools, Westmoreland County; West Chester School District, Chester County; and Franklin Street School, Elyria, Lorain County, Ohio.

In most of the schools the questionnaire was distributed by the elementary supervisor so that the children would perhaps feel freer to give a true answer and not merely the answer he thought would please his teacher. It was explained that the teacher would not see the answers, and because neither the child's nor the teacher's questionnaire asked for names, there was no pressure for approval or disapproval on certain questions. Grade placement and sex were indicated on the questionnaire. The elementary supervisor supplied such information as was necessary regarding the teacher's experience, training, and recent study in the field of arithmetic.

The composite results showed that 29 teachers and 1,023 students participated in the study. The graphs, charts, and summaries which follow describe the results of the study.

Tabulations of Questionnaire Data

The number of students involved in this study included 335 third grade students, 327 fourth grade students, and 361 sixth grade students, or a total of 1,023 students.

Each school's questionnaires were tabulated according to grade, then according to sex within the grade. Since there were no significant differences in the responses from the separate school districts, the tabulations were then combined to include the totals from each grade for all the school districts surveyed.

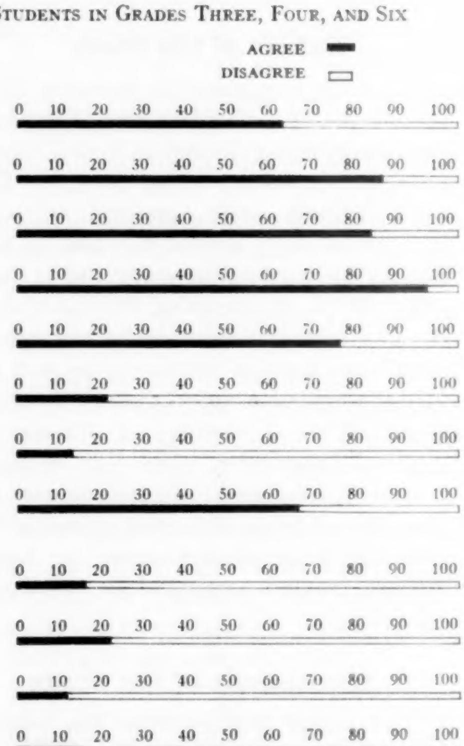
The attitude scale included 25 items to be checked as to whether the child agreed or disagreed with the statement. In the third grade the supervisor was instructed to read the statements if she felt the reading might prove too difficult for the youngsters, but she was instructed to read without comment as to meaning. The writer realizes that attitudes are not easily measured but felt nevertheless that this type of scale, if properly used, would yield a satisfactory measure, since there was no immediate and personal issue at stake. It will be recognized in the interpretation of the data that some error of measurement exists. However, the tabulations are from a fairly wide sampling and should offer stable results. The child was under no pressure as he might be in taking a test, for the questionnaire had no means of identifying him except by age, grade and sex. This was an effort to get an unbiased response from the child.

Responses to the 25 questions on the attitude scale are tabulated according to both sex and grade. In order to make comparisons of the responses, the data was also converted into percentages and shown by both grade and sex. The original tabular and graphic information is available with the original manuscript, on file at the Library, State College, Indiana, Pennsylvania.

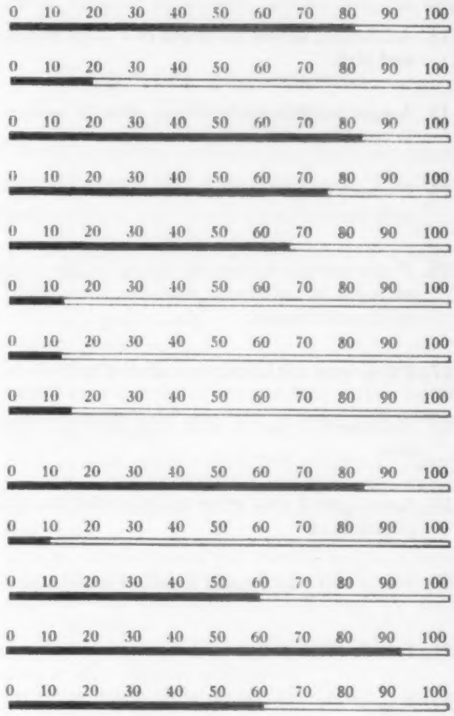
The first chart shows responses to the 25 items on the scale for all the students involved in the study. The second chart presents in graphic form the responses of the twenty-nine teachers to the thirty-five items on the teacher attitude scale.

RESPONSES ON ARITHMETIC ATTITUDE SCALE OF 1023 STUDENTS IN GRADES THREE, FOUR, AND SIX

1. If I had my way, everybody would study arithmetic.
2. Arithmetic is one of the most useful subjects I know.
3. All people should know arithmetic.
4. Arithmetic will help us in our daily lives.
5. Arithmetic has its faults, but I still like it.
6. Arithmetic is very uninteresting.
7. Nobody in our room likes arithmetic.
8. Arithmetic might be worth while if it were taught right.
9. Arithmetic is dull and boring.
10. I wouldn't take arithmetic if I didn't have to.
11. I don't even try to do my best in arithmetic.
12. I can't see how arithmetic will help me.

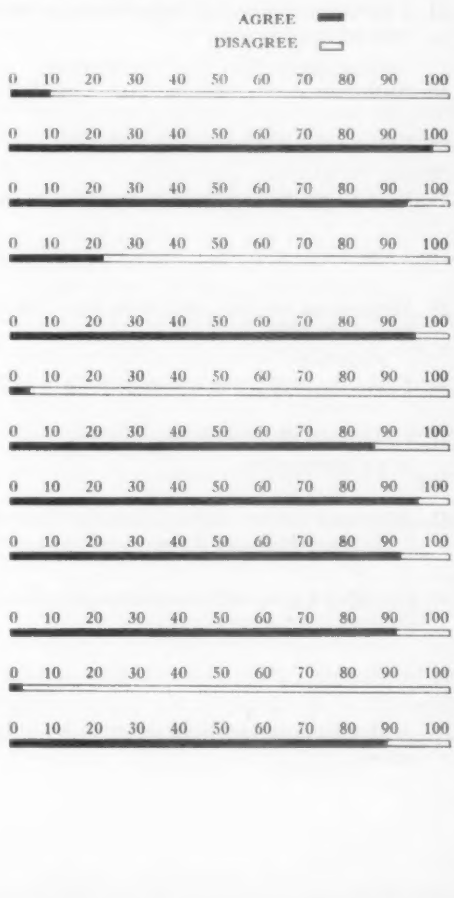


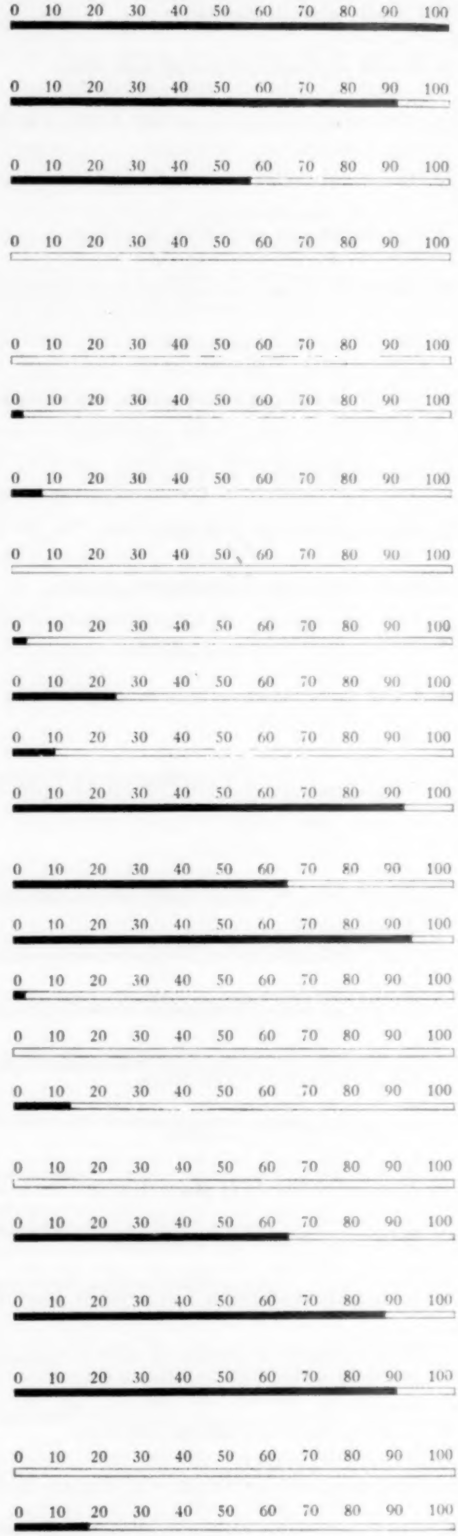
13. I really enjoy arithmetic.
14. I wish we'd miss arithmetic more often.
15. I like to figure and reason out problems in this class.
16. I've found arithmetic is useful at home.
17. I sometimes do extra work in arithmetic just for fun.
18. Arithmetic is just too hard for me to understand.
19. We get too much arithmetic.
20. I can't see how arithmetic will be very useful to me out of school.
21. Arithmetic teaches me to be accurate
22. Arithmetic is a waste of time.
23. Arithmetic is the best subject in school.
24. Arithmetic is okay.
25 I wish we had arithmetic more often.



Responses of 29 Teachers on Arithmetic Attitude Scale

1. No matter what happens, arithmetic comes first.
2. Arithmetic is of great value.
3. Arithmetic develops good reasoning ability.
4. I can teach arithmetic well without reading arithmetic magazines and methods books.
5. Arithmetic is very practical.
6. If some class has to be skipped, arithmetic is usually it.
7. I really enjoy teaching arithmetic.
8. Arithmetic is profitable to everybody who takes it.
9. I like to have the pupils try several ways of solving problems rather than to follow a suggested pattern.
10. I look forward to teaching arithmetic.
11. Arithmetic will benefit only the brighter students.
12. Arithmetic is one of the most useful subjects I know.





13. Arithmetic serves the needs of a large number of boys and girls.

14. A good teacher needs to keep up with modern methods in teaching arithmetic.

15. I spend more time on arithmetic than my schedule calls for.

16. If the teacher is sure she can solve the problems and exercises in arithmetic, she does not need to plan the arithmetic lessons.

17. I skip over arithmetic whenever I can.

18. Arithmetic is just a skill with little practical application.

19. Sometimes I give extra assignments in arithmetic as punishment.

20. I didn't like arithmetic in school and I still don't.

21. I can't seem to get arithmetic across to my students.

22. A good teacher follows the textbook very closely.

23. I give homework as a way of getting it across.

24. I use many devices and ways to get my students interested in arithmetic.

25. Arithmetic is very hard for the slow student.

26. I thoroughly enjoy teaching arithmetic.

27. I see no practical purpose in emphasizing arithmetic.

28. I wish I did not have to teach arithmetic.

29. Methods of teaching arithmetic have changed very little since 1930.

30. I have the feeling that my students hate arithmetic.

31. I always schedule arithmetic at a time when there will be no interruptions.

32. Arithmetic is never slighted. One time or another I fit it into my schedule each day.

33. I feel that I make arithmetic interesting to most of the children.

34. Arithmetic is the subject I like least of all to teach.

35. Arithmetic is not receiving the attention it deserves in school.

Summary of Data

Third grade. Since several studies seem to support the hypothesis that attitudes are formed as early as third grade, the following conclusions seem significant. However, one must take into consideration the lack of complete accuracy in third grade children's responses, part of which may be due to reading difficulty and part to the immaturity of the eight-year-old.

1. 19% of third grade children said arithmetic was dull and boring.
2. 20% thought arithmetic an uninteresting subject.
3. 25% said they wouldn't take arithmetic if they didn't have to.
4. 63% felt it was the best subject in school.
5. 65% sometimes do arithmetic just for fun.
6. 71% wished they had arithmetic more often.
7. 84% really enjoyed arithmetic.
8. 85% liked to figure and reason out problems.

Fourth grade. Study of the responses of fourth grade children to the attitude questionnaire seemed to justify the following statements. Again one must take into consideration inaccuracies due to lack of meaningful concepts.

1. 17% of fourth graders felt arithmetic was dull and boring.
2. 18% thought they got too much arithmetic.
3. 21% found arithmetic uninteresting.
4. 23% wouldn't study arithmetic if they didn't have to.
5. 63% wished they had arithmetic more often.
6. 79% really enjoyed arithmetic.
7. 86% liked to figure out problems.
8. 87% thought arithmetic one of the most useful subjects they knew.
9. 88% thought arithmetic helped them to be accurate.
10. 93% found arithmetic useful at home.

Sixth grade. After a study of the results of sixth grade children's responses, the following conclusions seemed significant.

1. 14% thought arithmetic dull and boring.
2. 14% felt they got too much arithmetic.
3. 18% found arithmetic uninteresting.
4. 53% thought it was the best subject in school.
5. 63% wished they had arithmetic more often.
6. 66% liked to do arithmetic for fun.
7. 81% really enjoyed arithmetic.
8. 85% liked to figure out problems.
9. 87% felt it taught them to be accurate.
10. 91% thought arithmetic was one of the most useful subjects in school.
11. 93% had found it helpful at home.
12. 98% thought arithmetic would help in our daily lives.

Summary Statements of Differences

From the tables and charts that have been tabulated from the data received, the following conclusions seem significant.

1. A small percentage in all three grades felt arithmetic was a waste of time.
   Third grade—13%
   Fourth grade—8%
   Sixth grade—6%
2. In third grade 23% couldn't see how arithmetic would be very helpful out of school, but in fourth grade this attitude was held by only 12% and in sixth grade 10%.
3. 25% of third graders and 23% of fourth graders said they wouldn't take arithmetic if they didn't have to, while only 13% of sixth graders gave this reply.
4. 61% of third graders felt that all people should know arithmetic, while by fourth grade this response had increased to 94% and by sixth grade to 98%.
5. In all three grades more than 80% of the children really enjoyed arithmetic.
6. In third grade 84% found arithmetic useful at home; in the fourth and sixth grades, 93% found it useful at home.
7. As for thinking arithmetic the best subject in school, 63% of the third graders agreed; 59% of fourth graders; and 53% of sixth graders.

Summary of All Students

When the attitudes of all the students were totaled from the data received from the attitude scale, the following conclusions seemed to be the most interesting and worthwhile:

1. Only 9% of the children checked felt that arithmetic was a waste of time.
2. 17% felt they got too much arithmetic.
3. 20% thought arithmetic uninteresting.
4. 58% said it was the best subject in school.
5. 66% wished they had more arithmetic.
6. 69% liked to do extra arithmetic just for fun.
7. 81% said they really enjoyed arithmetic.
8. 86% classified arithmetic as the most useful subject in school.
9. 87% felt that arithmetic taught them to be accurate.
10. 95% of all the children felt that arithmetic would help them in their daily lives.

Comparisons Between Boys and Girls

Since studies show that boys tend to achieve better scholastically in arithmetic, the results of the questionnaire made for an

interesting comparison between the attitudes of boys and girls.

1. 20% of both boys and girls felt arithmetic was an uninteresting subject.
2. 54% of the boys and 63% of the girls thought arithmetic was the best subject in school.
3. 61% of the boys and 71% of the girls wished they had arithmetic more often.
4. 65% of the boys and 73% of the girls did extra arithmetic just for fun.
5. 76% of the boys and 84% of the girls stated that they really enjoyed arithmetic.
6. 76% of the boys and 94% of the girls felt everyone needed to know arithmetic.
7. More than 85% of both boys and girls agreed that arithmetic was one of the most useful subjects they knew.

Summary and Conclusions Drawn from the Teachers' Data

It has already been stated elsewhere in this study that the teachers who completed the attitude scale were not identified by name. It was felt that perhaps this would tend to present a truer picture than if the teacher were under pressure to give the answer he felt was wanted or that he knew was acceptable in the light of present-day methods of teaching. The elementary supervisor did supply enough information so that the writer might ascertain whether or not the teacher's age, training, experience, and recent study made any significant difference in attitude. The years of experience ranged from thirty-two years to one. Several teachers were near retirement age while others were recent college graduates. Of the teachers surveyed fourteen held a Bachelor of Science degree, six had the Master of Education degree, and nine no degree and no recent college work. The writer realizes that true attitudes are very difficult to measure. However, from the data tabulated the following conclusions may be significant.

1. 90% of all teachers said that no matter what happens, they fit arithmetic into their schedule each day.
2. 93% stated that they really enjoy teaching arithmetic, while 97% indicated that they thoroughly enjoy teaching arithmetic. Several questions were repeated in different form as a check. This was one which varied slightly.
3. 90% of teachers felt that a good teacher should keep up with modern methods, but 21% felt they could teach arithmetic well without reading periodicals and methods books.
4. 17% felt that methods of teaching arithmetic had not changed in the past thirty years.
5. All of the teachers agreed that arithmetic is of great value.

Conclusions and Implications

After analyzing the results of the study of pupil attitudes by grade and by sex, as well as the teachers' responses, the writer draws the following conclusions and implications:

1. A large percentage of elementary teachers really enjoy teaching arithmetic and use many devices to make it interesting.
2. All teachers felt that lessons should be carefully planned. It was not felt sufficient that the teacher just know how to solve the problems and exercises. This does not imply that all of these teachers do plan each arithmetic lesson carefully, but that they know the value of careful planning.
3. The teacher's educational background, recent training, age, or years of experience seemed to make no significant difference in his attitude toward the teaching of arithmetic, nor of the attitude of the children in the group.
4. Contrary to popular opinion, a very large percentage of both boys and girls do like arithmetic and feel it is a very useful subject.
5. The majority of the teachers answering the questionnaire really enjoy teaching arithmetic. This does not imply that all of the teachers are good teachers of arithmetic.
6. Definite attitudes in students have developed by the third grade.
7. As children grow older they develop more meaningful concepts in arithmetic.
8. According to previous studies, girls tend to make consistently better scores than boys in general studies. Boys seem to have a slight edge in arithmetic, geography and science. However, in all three grades measured in this study, girls liked arithmetic better than boys, and 22% more of the girls thought everyone should know arithmetic. No attempt has been made to check the correlation between arithmetical achievement and attitude toward arithmetic.

EDITOR'S NOTE. As the author frequently remarks, this is an area in which measurement is not easy. Taken at face value, does the decline from grade to grade in the per cent of students expressing unfavorable reactions to arithmetic indicate a real change, or is it sophistication, or merely acceptance of verbalisms to the effect that it is good for them?

Conclusion 3 in the writer's summary, especially the last part thereof, does not seem to be supported by the data shown in the article. Conclusion 8 poses an interesting paradox. It seems likely that the correlation between attitudes found here and arithmetical achievement, if examined, might turn out to be negative. What would this mean?

A Bibliography of Historical Materials for Use in Arithmetic in the Intermediate Grades

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THE USE OF HISTORICAL materials in the teaching of arithmetic stimulates interest, enthusiasm, and appreciation among pupils for the learning of arithmetic. These materials may also aid pupils to see certain characteristics and relationships. Arithmetic textbooks cannot present complete historical stories on the development of the number system, the number processes and the system of measures. It is important that teachers have available some supplementary books which are suitable for use with children and which can serve as sources of such historical information. Of course, encyclopedias are also sources of some historical information about arithmetic, but these have not been included in the bibliography presented.

As an illustration of the use of historical materials with children, the following description is presented. A fifth grade class is extending the study of multiplication of whole numbers to multiplication by a three-place multiplier. They have examined the meaning of multiplying by each digit in the multiplier and obtaining partial products. For example, the multiplication involved in

$$\begin{array}{r} 327 \\ \times 243 \\ \hline \end{array}$$

has been expressed as

$$\begin{array}{r} 327 \\ \times 3 \\ \hline 981 \end{array}, \quad \begin{array}{r} 327 \\ \times 40 \\ \hline 13080 \end{array}, \quad \text{and} \quad \begin{array}{r} 327 \\ \times 200 \\ \hline 65400 \end{array},$$

and the partial products added to secure the total product.

$$\begin{array}{r} 981 \\ 13080 \\ 65400 \\ \hline 79461 \end{array}$$

The pupils have also performed the multiplication in the shorter, more conventional manner.

$$\begin{array}{r} 327 \\ \times 243 \\ \hline 981 \\ 1308 \\ 654 \\ \hline 79461 \end{array}$$

As a possible means of stimulating pupils to extend their skill in multiplication of whole numbers, to appreciate arithmetic from a historical point of view, and to study further the meaning and placement of partial products, besides introducing an interesting device for checking multiplication, the fifth grade teacher then reads the children a short story about "computing rods."

A brief story about this topic may be found in the enrichment booklet *Ways to Multiply*, by Harold D. Larsen, published by Row Peterson and Company. The story is entitled "Computing Rods." It tell us that computing rods made of wood or ivory were very popular for use in multiplying during the seventeenth century. The teacher

may add the information that these rods were called "Napier's Bones," because the plan was invented by a Scottish mathematician, John Napier, who lived from 1550 to 1617.

How children may make "computing rods" from cardboard and use them to multiply is also explained in the story. A diagram like that shown in Figure 1 may be drawn. This diagram would then be cut into strips along the vertical lines.

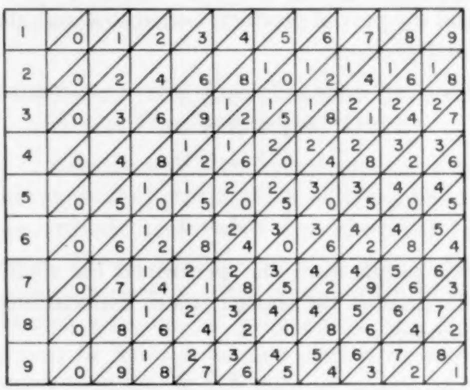
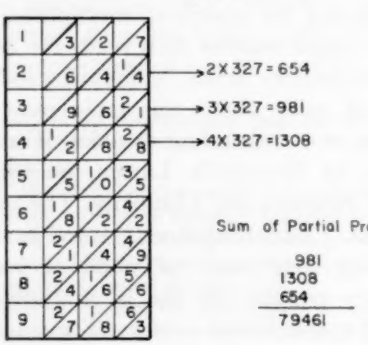


FIG. 1

In using these computing rods to check the example shown, the pupil would put the rods together as in Figure 2 and write the partial products. The "index" strip is first placed to the left. Since the multiplicand of the example being checked is 327, the rods with 3, 2, and 7 at the top would then be placed in the order shown in Figure 2.

FIG. 2



The proper placement of these partial products requires the pupil to understand that the partial product given beside the index figure must be multiplied by 10 (moved one place to the left) if this figure is in the tens place in the multiplier of the example being used. Therefore the numeral 1308 stands for 13080 when it is positioned one place to the left in the placement of partial products, and the zero isn't needed. If the index figure represents a figure in the hundreds place as does 2, then the partial product is multiplied by 100 by moving it two places to the left as shown. This placement of partial products should be compared with the placement of partial products in the conventional manner of multiplying. The reading of this story and carrying out the activity as described should stimulate considerable interest in multiplication on the children's part.

Bibliography

The books listed in the following bibliography were selected for use mainly at the intermediate grade level. This list was compiled with consideration given to the problems faced by the classroom teacher in the average classroom situation where abilities and interests cover a wide range. It was judged necessary to include easy materials on a primary grade reading level as well as more difficult materials on the junior high reading level. Some of the stories included in the bibliography can be used for independent reading by pupils. In addition, it is recommended that the teacher frequently read aloud from these books to the pupils. It is from this point of view that the grade levels are suggested.

History and Development of the Number System

ADLER, IRVING, *Mathematics*, 56 pp., Golden Press, 1958, $.50, Grade 6–adult.

Mathematics is brought from its historical beginning to an explanation of present-day uses. Reasons are given for the invention of arithmetic, geometry, trigonometry, and algebra.

ANDERSON, RAYMOND W., *Romping through Mathematics*, 152 pp., Knopf, 1947, $3.00. Grades 6–9.

This history of mathematics from arithmetic to integral calculus interprets ideas and relationships with examples from everyday life.

ASIMOV, ISAAC, *Realm of Numbers*, 200 pp. Houghton, 1959. $2.75, Grades 6–11.

Historical facts include explaining uses of the abacus and basic procedures in mathematics.

FRIEND, NEWTON, *Numbers: Fun and Fact*, 208 pp., Scribner, 1954, $2.75. Grades 6 and up.

Mathematical oddities, curiosities, puzzles, and problems are included. Readers learn how the number system and various mathematical concepts started.

HOGBEN, LANCELOT THOMAS, *The Wonderful World of Mathematics*, 79 pp., Garden City, 1955, $2.95. Grades 5–9.

The progress of mathematics from man's earliest methods of counting and measuring time helps to show how men used mathematics in the various aspects of their lives.

IRWIN, KEITH GORDON, *The Romance of Writing*, 160 pp. Viking, 1956, $3.75. Grades 3–10.

Story of writing from caves of early tribes through evolution of our numbers and signs. Approximately 100 illustrations are included.

LARSEN, HAROLD D., *Enrichment Program for Arithmetic*, 15 pp. per booklet (set of 8 booklets for each grade—3, 4, 5, 6), Row Peterson, 1956, $1.60.

"The Story of Counting Beyond Ten" (Grade 3)—This is the story of how the idea of counting by groups of ten and ones came into use. It presents an interesting discussion of the origin of the words eleven and twelve.

"The Story of Zero" (Grade 4)—In this booklet zero tells how important he is as a placeholder in our number system. Also explained is the fact that zero is a starting point on thermometers, rulers, and measuring cups.

"Excursions in Arithmetic" (Grade 6)—The first part of this booklet shows quick addition and triangular numbers used by Greeks. We are told how early farmers plowed with oxen in the morning and in the afternoon measured their land to see how much they had plowed. Use of fractions began because man found need to use part of things.

PINE, TILLIE S., and LEVINE, JOSEPH, *The Chinese Knew*, 32 pp., McGraw, 1958, $2.50. Grades 1–4.

Described are many things which the Chinese have used for thousands of years, including the abacus and compass.

SMITH, DAVID EUGENE, *Number Stories of Long Ago*, 136 pp., reprinted by NCTM, $1.00. Grades 5 and up.

"How Ching and An-Am and Menes Counted" (pp. 1–12)—Here is explained how people once counted only as far as two, three, or four and how finally our numbers developed beyond the simple groups used by primitive men.

"How Ahmes and Lugal and Chang Wrote Their Numbers" (pp. 13–21)—Story of how three Chinese, Babylonian and Egyptian boys all learned to write numbers. The Chinese painted marks on palm leaves; the Babylonians marked with sticks on clay; and the Egyptians progressed from writing on stone and brick to papyrus. Illustrations show that many of the symbols for different figures were very much alike.

"How Hippias and Daniel and Titus Wrote Their Numbers" (pp. 23–31)—Some of the first types of paper were parchment rolls made from sheepskins. Together with the problem of writing materials, the Greek, Hebrew, and Roman people had the problem of learning certain symbols to express number.

"How Gupta and Mohammed and Gerbert Wrote Their Numbers" (pp. 33–43)—Tells how three boys learned about numbers. In India Gupta learned only four numbers; Mohammed learned the Hindu numbers from the Arabs; and the French boy, Gerbert, was sent to Spain to learn more about numbers. Points up how number knowledge slowly spread.

"Ahmes and Heron and Jakob Despair of Ever Learning Fractions" (pp. 83–91)—Tells how beginning fractions all had numerators no greater than one, while two thousand years later all fractions had 60 for denominators. Later, in Germany, our present system came into being.

SMITH, DAVID EUGENE, *The Wonderful World of One-Two-Three*, 47 pp., McFarlane, 1937, $1.00. Grades 3 and up.

This history of our number symbols gives an account of naming of numbers and their early use.

SMITH, DAVID EUGENE, and GINSBURG, JEKUTHIEL, *Numbers and Numerals*, 52 pp., NCTM, 1937, $.35. Grade 5 and up.

"Learning to Count" (pp. 1–3)—Story suggests that our lives might have been greatly changed had people not learned to use numbers. Points out many ways in which we depend on number. Also suggested are uses of number bases other than ten.

"Naming the Numbers" (pp. 4–6)—Presents ways numbers were named in other languages and points to various spelling endings which some languages use. An explanation is given for the names of the last four months of the year.

"From Numbers to Numerals" (pp. 7–24)—Reader is taken back to beginnings of number thinking and usage. Gradual development from number words to use of numerals is shown.

"Fractions" (pp. 35–38)—Explanation of why fractions were more difficult to learn in ancient times, since no one knew an easy way to write them. Also given are some uses of fractions and old and recent ways of writing them.

"Mystery of Numbers" (pp. 39–43)—Much of superstition and mystery which men have attached to number in both Biblical and historical events is revealed.

History and Development of the Number Processes

From Abacus to Monroe, 16 pp., Monroe, 1930, free. Grade 6 and up.

Calculating machine history is discussed, from the simple abacus to complicated machines constructed to add, subtract, multiply, and divide.

From Og to Googol, 13 pp., Marchant, 1946, free. Grade 5 and up.

This brief history of numbers begins with simple counting or adding on fingers and progresses to use of modern calculators.

LARSEN, HAROLD D., *Enrichment Program for Arithmetic*, 15 pp. per booklet, set of 8 booklets for each grade (3, 4, 5, 6), Row Peterson, 1956, $1.60.

"Ways to Multiply" (Grade 5)—This booklet includes a story of how Ethiopians multiply. Also discusses finger multiplying, lattice multiplication, the scratch method, and computing rods.

SMITH, DAVID EUGENE, *Number Stories of Long Ago*, 136 pp., reprinted by NCTM, $1.00. Grade 5 and up.

"How Robert and Wu and Caius Added Numbers" (pp. 45–61)—The story explains the difficulty of adding with Roman numerals and how pupils learned.

"How Cuthbert and Leonarda and Johann Multiplied Numbers" (pp. 63–71)—Through experiences of the three boys we see how multiplication problems were first written and how the world tried the use of multiplying.

"How Flippo and Adriaen and Michael Divided Numbers" (pp. 73–81)—Our method of dividing developed from the "scratch method" and "galley" method. From examples of older methods of division an explanation is given for the long period before our method of dividing was accepted.

SMITH, DAVID EUGENE, and GINSBURG, JEKUTHIEL, *Numbers and Numerals*, 52 pp., NCTM, 1937, $.35. Grade 5 and up.

"From Numerals to Computation" (pp. 25–34)—Story tells how people began to use numbers for adding, subtracting, multiplying, and dividing. From simple machines, such as the abacus, modern business machines had their origins. Much of this story deals with the use of the abacus and how addition, subtraction, and multiplication were main arithmetical processes.

History and Development of Measures

ADLER, IRVING, *Time in Your Life*, 127 pp., Day, 1955, $2.75. Grades 5–9.

Many fields of knowledge are considered which need time as a basis for understanding. Among these are: man's first timepiece, stars and planets, rocks and rivers, history and calendar, time zones, music and dancing, and others. Illustrations aid reader in understanding the text. Directions for a perpetual calendar are included.

BENDICK, JEANNE, *How Much and How Many, The Story of Weights and Measures*, 188 pp., McGraw, 1947, $2.75. Grades 5–9.

How weights and measures have affected trade, science, and everyday living.

BRINDZE, RUTH, *The Story of Our Calendar*, 64 pp., Vanguard, 1949, $2.50. Grades 4–8.

How the calendar got its name, how "leap year" was started, and ways of telling time around the world are explained. Also told is how different calendars through the ages were developed to keep track of time according to sun, moon, and stars.

EISENBERG, AZRIEL LOUIS, *The Story of the Jewish Calendar*, 62 pp., Abelard, 1958, $2.50. Grades 3–6.

The difference between solar and lunar calendars, how dates of religious holidays are set, how Jewish months were named, different names for Sabbaths throughout the year and their significance, names and times of religious holidays, and the proposed world calendar and its significance are explained.

GALT, TOM, *Seven Days from Sunday*, 215 pp., Crowell, 1956, $3.00. Grades 5–9.

The story is told of the many people and cultures which contributed to our present system of days and weeks. Here are also Teutonic legends of Thor, Woden, and Tiu, and the lesser-known but astonishing story of Mithras, god unconquered by the sun. Names of the days are listed in many languages so that reader can compare them.

How Long Is a Rod? 6 pp., Ford, free. Grades 5 and up.

Described are the origin of our standard of length and the necessity of precise methods of measurement as a forerunner to development of modern mass production.

LARSEN, HAROLD D., *Enrichment Program for Arithmetic*, 15 pp. per booklet (Set of 8 booklets for each grade—3, 4, 5, 6), Roy Peterson, 1956, $1.60.

"The Story of Measure" (Grade 5)—This booklet presents a story of how man came to have a need for measure and how he tried many things before standard measures were set up. The need for standard measures is explained.

"The Story of Money" (Grade 5)—Among the facts included in the booklet are how Indians first traded furs with white men for supplies, how pygmies still use "silent barter," how children barter or trade articles, and how barter and trading do not always work. Money or coins were found easier to carry and divide than earlier forms of currency.

"The Story of Time" (Grade 4)—Story shows that practically everything we do is done by the clock. Descriptions are given for timepieces beginning with the sun and continuing to present-day suggestions for "atomic clock."

LEEMING, JOSEPH, *From Barter to Banking*, 131 pp., Allyn, 1940, $2.00. Grades 4–7.

Factual materials with historical background are used in developing story of money from barter to banking. There are descriptions of many different kinds of money that have been used and are still being used.

MASTERS, ROBERT V., and REINFELD, FRED, *Coinometry*, 93 pp., Sterling, 1958, $3.50. Grades 5–7.

Presents a historical introduction to coins and currency, with up-to-date catalog values and 363 photographs.

STERLING, DOROTHY, *Wall Street: The Story of the Stock Exchange*, 128 pp., Doubleday, 1955, $2.75. Grades 6–9.

Tells the history of present-day functioning of the stock market.

TANNENBAUM, BEULAH, and STILLMAN, MYRA, *Understanding Time: The Science of Clocks and Calendars*, 144 pp., Whittlesey, 1958, $3.00. Grades 6 and up.

Explains value and meaning of telling time with clocks and watches, sundial, hourglass, and water clocks. Tells how people used moon, sun, and stars to mark passing days, months, and years. Related science experiments are included.

The Amazing Story of Measurement, 23 pp., Lufkin, 1959, free. Grades 5–7.

Story of measurement from cavemen to precision methods used today is told in comic-book form. Brief explanation is given for each of the following measures of length: cubit span, palm, digit, foot, meridian mile, fathom, reed or rod, mile, inch, yard, ell, pace, furlong, and hand.

ZARCHY, HARRY, *Wheel of Time*, 114 pp., Crowell, 1957, $2.75. Grades 6–8.

Development of man's ideas about time and evolving of year, months, days, hours, and seasons are explained. Also described are curious types of time-keepers.

ZINER, FEENIE, and THOMPSON, ELIZABETH, *The True Book of Time*, Children's Press, 1956, $1.50. Grades 1–4.

Various ways that have been used to measure time are described.

Directory of Publishers

ABELARD. Abelard-Schuman, Ltd., 404 Fourth Avenue, New York 16, New York.

ALLYN. Allyn and Bacon, Inc., 70 Fifth Avenue, New York 11, New York.

CHILDREN'S PRESS. Children's Press, Inc., Jackson Blvd. and Racine Avenue, Chicago 7, Illinois.

CROWELL. The Thomas Y. Crowell Company, 432 Fourth Avenue, New York 16, New York.

DAY. John Day Company, Inc., 210 Madison Avenue, New York 16, New York.

DOUBLEDAY. Doubleday and Company, Inc., Institutional Department, Garden City, New York.

FORD. Ford Motor Company, The American Road, Dearborn, Michigan.

GARDEN CITY. Garden City Books, 575 Madison Avenue, Garden City, New York.

GOLDEN PRESS. Golden Press, Inc., Rockefeller Center, New York 20, New York.

HOUGHTON. Houghton Mifflin Company, Boston, Massachusetts.

KNOPF. Alfred A. Knopf, Inc., 501 Madison Avenue, New York, New York.

LUFKIN. The Lufkin Rule Company, Saginaw, Michigan.

MCFARLANE. McFarlane, Warde, and McFarlane, Inc., 213–215 Cayuga Street, Fulton, New York.

MCGRAW. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, New York.

MARCHANT. Marchant Calculators, Inc., 1475 Powell Street, Oakland 8, California.

MONROE. Monroe Calculating Machine Company, Inc., P. O. Box 540, Orange, New Jersey.

NCTM. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.

ROW PETERSON. Row Peterson and Company, 2500 Crawford Avenue, Evanston, Illinois.

SCRIBNER. Charles Scribner's Sons, 597 Fifth Avenue, New York 17, New York.

STERLING. Sterling Publishing Company, Inc., The Sterling Building, 121 East 24th Street, New York 10, New York.

VANGUARD. Vanguard Press, 424 Madison Avenue, New York 17, New York.

VIKING. Viking Press, Inc., 625 Madison Avenue, New York 22, New York.

WHITTLESEY. Whittlesey House, 330 West 42nd Street, New York 36, New York.

EDITOR'S NOTE. The belief on the part of many students and some teachers that arithmetic has been handed down, intact, from on high, might well be modified by the early use of materials closer to the sources. Such material as that listed by the authors might be expected to yield this as an added dividend over and above the motivating effect. It is hoped that the authors' estimates as to the best grade ranges will be a useful addition to the bibliography.

Admirable Numbers and Compatible Pairs

J. M. SACHS
Chicago Teachers College, Chicago, Illinois

THE OBJECT OF THIS PAPER is to suggest an extension of the number theory concepts of *perfect numbers* and *amicable pairs* of numbers. The suggested extension can provide interesting material for practice with the basic operations with integers. In addition to serving as a medium for this necessary practice, this extension yields an area in which children can explore and make discoveries. In their explorations they will see mathematics as a subject of ideas in which intuition, imagination, and speculation play large roles. This is an aspect of this discipline which is not exhibited early enough or often enough, probably because we, as teachers, find ourselves so greatly concerned with teaching the necessary techniques.

If the sum of all the proper* integral divisors of a positive integer is equal to that integer, the integer is called a *perfect number.* The proper divisors of 6 are 1, 2, and 3. The sum of 1, 2, and 3 is 6. The proper divisors of 28 are 1, 2, 4, 7, and 14. The sum of 1, 2, 4, 7, and 14 is 28. *Perfect numbers* are relatively rare among the integers. Among the known *perfect numbers* are 6, 28, 496, and 8128.

If the sum of all the proper integral divisors of a positive integer is less than that integer, the integer is called a *deficient number.* The proper divisors of 8 are 1, 2, and 4. The sum of 1, 2, and 4 is 7, which is less than 8. The proper divisors of 9 are 1 and 3. The sum of 1 and 3 is 4, which is less than 9. The proper divisors of 15 are 1, 3, and 5. The sum of 1, 3, and 5 is 9, which is less than 15. Thus 8, 9, and 15 are *deficient numbers.*

If the sum of all the proper integral divisors of a positive integer is greater than that integer, the integer is called an *abundant number.* The proper divisors of 12 are 1, 2, 3, 4, and 6. The sum of 1, 2, 3, 4, and 6 is 16, which is greater than 12. The proper divisors of 18 are 1, 2, 3, 6, and 9. The sum of 1, 2, 3, 6, and 9 is 21, which is greater than 18. The proper divisors of 24 are 1, 2, 3, 4, 6, 8, and 12. The sum of 1, 2, 3, 4, 6, 8, and 12 is 36, which is greater than 24. Thus 12, 18, and 24 are *abundant numbers.*

The extension of the concept of *perfect numbers* to a larger set which I shall call *admirable numbers* is part of some material I developed for a television in-service training course in mathematics for elementary school teachers. Since that time I have had the opportunity of trying out this material on several classes of elementary school children.

If a positive integer can be expressed as an algebraic sum of all of its proper integral divisors, let us call it an *admirable number.* (In introducing this concept to children I did not, of course, use the words "algebraic sum." I suggested that we wanted to add some of the divisors and subtract all of the others from this sum.)

EXAMPLES:

	Proper Divisors	
12	1, 2, 3, 4, 6	$12=1+3+4+6-2$
24	1, 2, 3, 4, 6, 8, 12	$24=4+6+8+12-1-2-3$
42	1, 2, 3, 6, 7, 14, 21	$42=1+2+3+7+14+21-6.$

Thus 12, 24, and 42 are *admirable numbers.* It is obvious that we must look for the *non-perfect admirable numbers* among the *abundant numbers.* It can also be easily seen by checking a few small *abundant numbers* that not all of them are *admirable.* It is not possible to get 18, for instance, as the algebraic sum of its proper divisors.

* A proper divisor of a positive integer is any positive integral divisor except the original integer itself.

If we refer again to classical number theory, we find that a pair of positive integers is called an *amicable* pair if each is equal to the sum of all of the proper divisors of the other. An example is the pair consisting of 220 and 284. The proper integral divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110. The sum of all of these divisors is 284. The proper integral divisors of 284 are 1, 2, 4, 71, and 142. The sum of all of these divisors in 220. *Amicable* pairs involve rather large numbers and so present some difficulties for children. Among the other known pairs are (1184; 1210) and (17296; 18416).

I propose that we call two integers a *compatible pair* if each can be expressed as an algebraic sum of all of the proper integral divisors of the other.

In the experiences I have had in presenting this material to elementary school children, I have found that they were very much interested in number theory and quite capable of understanding it. The average groups showed the same zest for exploration and discovery, though they did not produce as many generalizations as the accelerated groups. There are a number of obvious and some not so obvious advantages gained in using this material. As previously mentioned, this is a rather novel framework for a review and for practice with the basic operations in the arithmetic of whole numbers. The child must do division to find the proper integral divisors of the given number. In this he can be guided to notice that whenever he finds one divisor, the quotient obtained upon division by this divisor is itself a divisor. For example when he discovers that 3 is a divisor of 42, he has discovered that 14 is also a divisor of 42 since $42 \div 3 = 14$. If he investigates this further, he discovers that proper integral divisors occur in pairs with two exceptions. With the divisor 1 is paired the given integer, which is not a proper divisor of itself. Also, if the given integer is the square of another integer, then dividing by the square root of the given integer will yield, as quotient, not another divisor but the same square root.

This number theory material can also provide for a review of multiplication since in looking for the divisors of 42, for example, the child can be encouraged to ask such questions as, "Two times what number gives 42 as product?" and "Three times what number gives 42 as product?" and thus not only review multiplication but see division as the inverse of multiplication. The use of this material in a review of basic addition and subtraction is apparent, and it can also serve as a foundation for the addition of signed numbers.

The advantages which are perhaps not obvious in using this material center around the ideas of exploration and discovery in mathematics. This is new material in a very real sense. The child will not find a table of *admirable numbers* or *compatible* pairs in any text or reference book. He must, if his curiosity is aroused, investigate for himself. He can be given the thrill of discovery particularly since, to my knowledge, these areas have not been previously explored. He can examine number relationships and try to make generalizations. The children who helped try out this material enjoyed this aspect very much. They seemed to consider looking for generalizations as a wonderful game and were not in the least bit

EXAMPLES:

Number Pair	*Divisors*	
30	1, 2, 3, 5, 6, 10, 15	$2+3+5+6+10+15-1=40$
40	1, 2, 4, 5, 8, 10, 20	$1+2+4+5+8+20-10=30$

Thus the pair (30; 40) is a *compatible* pair.

42	1, 2, 3, 6, 7, 14, 21	$2+3+6+7+14+21-1=52$
52	1, 2, 4, 13, 26	$1+4+13+26-2=42$

Thus the pair (42; 52) is a *compatible* pair.

self-conscious about this kind of activity. We spent some time testing speculations, or "guided guesses" as the children called them. This last activity I feel is an important one in establishing a foundation for later efforts to "prove" generalizations.

In conclusion I would like to suggest a number of generalizations and conjectures, many of which came as the result of the speculative and testing activities of the children.

1. We look for *admirable numbers* only among the *abundant numbers*.
2. Some *admirable numbers* can be expressed as sums and differences of all proper divisors in more than one way.
3. Not all *abundant numbers* are *admirable*.
4. If a number is *admirable*, then the difference between the number and the sum of all of its proper divisors is even.
5. There are odd *admirable numbers*.
6. The smallest odd *admirable number* is 1575.
7. Any integral multiple of a *perfect number* is *abundant*.
8. The smaller number of an *amicable* pair or a *compatible* pair must be abundant, and the larger number of an *amicable* pair must be *deficient*.
9. There are *compatible* pairs with the larger number *deficient*.
10. If the sum of the divisors of one member of a *compatible* pair is even, the other member of the pair is an even number.

EDITOR'S NOTE. Don't dismiss this article with a "What's this, back to the Greeks?" attitude. Easy material that lends itself to real discovery and to the development of the power to generalize is scarce. Some of the experimental programs now underway are also turning up indications, similar to the experience of Professor Sachs, that children enjoy this sort of thing. They also appear to profit by it even when judged by conventional tests.

From the Editor's Desk—

We share with you a letter received with reference to Dr. Brownell's recent article in the April, 1960, issue of THE ARITHMETIC TEACHER

Dear Sir,

I was interested to read William Brownell's article "Observations on Instruction in Lower-Grade Arithmetic in English and Scottish Schools," contained in the April issue of THE ARITHMETIC TEACHER.

It was a great privilege to meet Dr. Brownell and to discuss with him some of the newer work which is now taking place in our schools. However, I think I should attempt to correct the impression that Dr. Brownell has given that in the school he visited with me a great deal of time is spent on arithmetic. Actually the school concerned spends on average, no more than 45 minutes per day on this subject. On the occasion of Dr. Brownell's visit, everybody was so interested that the lesson went on for very much longer than was usual. Another point, which is not made so loudly, but is nevertheless implied, is that the teachers are specially gifted. This also is not so, although, of course, they have certainly become more interested as their work has progressed.

One point that has emerged since the occasion of William Brownell's visit has been the fact that children do seem to learn mathematical ideas very much more easily as a result of handling structural material which has been specially designed to enable children to learn the mathematical ideas in an individual way.

Yours truly,

L. Y. W. SCULEY
Adviser for Junior Schools
County of Leicester

New Devices Elucidate Arithmetic

MILTON W. BECKMANN*
University of Nebraska
Lincoln, Nebraska

THE NATIONAL DEFENSE EDUCATION ACT of 1958, briefly referred to as NDEA, authorized something over one billion dollars in Federal aid over a four-year period. "In the swinging sweep of its 10 titles it touches—and returns to touch again—every level of education, public and private, from the elementary school through the graduate school. Its billion dollars, though authorized for a dozen separate programs, have been authorized for a single purpose—that every young person, from the day he first enters school, should have an opportunity to develop his gifts to the fullest."[1] It is quite clear that Congress recognized how exceedingly important is superior instruction at the elementary level as well as in high school and college. The Act concerns itself with the finding and encouraging of talent, the improving of teaching, and with the furthering of knowledge itself. This Act includes the instructor of arithmetic.

Financial assistance for strengthening science, mathematics, and modern foreign language instruction is offered through Title III of the NDEA. Under this Title, Congress authorized seventy million dollars each year for four years for science, mathematics and foreign language equipment and minor remodeling of laboratory or other space that would make such equipment usable. Congress did not appropriate the maximum amount authorized during the first two fiscal years for this phase of the program. The actual appropriations for acquisitions were \$56,000,000, and \$60,000,000 in 1959 and 1960 respectively.[2] For state supervisory and related services, \$5,000,000 were authorized each year for four years. The actual appropriations were \$1,350,000 and \$4,000,000 for supervisory and related services, in 1959 and 1960 respectively.

Through NDEA, an exceptional opportunity is presented to the arithmetic teacher. The imaginative teacher will be able to strengthen instruction in arithmetic through the use of aids such as abacuses, place value charts, dial clocks, geometric solids, chalk board compasses, counting frames, flash cards, fraction disks, English liquid measures, metric liquid measures, and geometry ruler sets.

The *Purchase Guide for Programs in Science, Mathematics, and Foreign Language*, prepared by the Council of Chief State School Officers, contains a list of equipment for the elementary teacher in mathematics.[3] Although there are a few items listed which are not eligible for purchase under NDEA, every instructor in arithmetic should have access to this helpful guide. Most administrators have a copy. Items are alphabetically arranged and described, with specifications and categorizations, such as elementary and secondary and standard or additional, given for each item. Below are several items found in the guide.

* Dr. Beckmann worked in the United States Office of Education, Department of Health, Education and Welfare, for one year as a Specialist in Mathematics under the National Defense Education Act. He is now Supervisor of Mathematics and Professor of Education at the University of Nebraska.

[1] Office of Education, *Guide to the National Defense Education Act of 1958*, rev. ed. (Washington, D. C.: U. S. Department of Health, Education, and Welfare, 1959), p. 1.

[2] Twelve per cent of these sums was set aside for loans to private schools.

[3] Council of Chief State School Officers, *Purchase Guide for Programs in Science, Mathematics, and Foreign Language* (Ginn and Company, 1959).

Abacus, Computing

Mathematics, Elementary—Standard, One for Each Classroom

A manipulative device to aid elementary school pupils in developing an understanding of number concepts, verifying arithmetic processes, etc.

Specifications: The computing abacus shall consist of a horizontal base with an opaque, upright and parallel center portion over which four wires shall be arched at evenly spaced intervals with the ends of each mounted firmly in the base. Each wire shall carry 20 disks except the thousands wire which shall have 9 disks. Space dividers shall be provided to permit the user to carry-over in addition and multiplication, and to regroup in subtraction. A decimal point insert shall also be supplied to permit computation with decimals. A complete manual of instructions shall be included.

Figures, Plane Geometry, Demonstration Set

Mathematics, Elementary—Standard, One Set for Each School

Mathematics, Secondary—Standard, One Set for Each Classroom

Series of manipulative sensory aids for demonstrating the basic principles of plane geometry and to guide the students' learning by induction. An aid in discovering relationships intuitively.

Specifications: The construction board set shall consist of ten boards with complete instructions for use. The boards shall be made of sturdy pressed wood, each equipped with elastics and pegs for quick and easy manipulation. Each set shall include the individual purchase, when specified:

1. Triangles and their angles;
2. Quadrilaterals and parallelograms and their diagonals;
3. Polygons;
4. Pythagorean theorem;
5. Altitudes of triangles;
6. Circle circumscribed about a triangle;
7. Triangles equal in area;
8. Angles inscribed in a semicircle;
9. Medians of triangles;
10. Perpendicular bisector.

Another helpful source for the elementary mathematics teacher is *A Guide to the Use and Procurement of Teaching Aids for Mathematics*,[4] published by the National Council of Teachers of Mathematics. The first part of the guide deals with the use of teaching aids, functions of teaching aids, how to use teaching aids effectively, and suggested criteria for selection of equipment and materials. The remainder of the publication deals with representative lists of aids, their sources of supply, and in some cases the costs that should be anticipated.

Mathematical relationships are more easily understood and retained if they are illustrated with diagrams and models. The mathematical principles which govern the operation of fractions are seldom understood by pupils. If this understanding of the basic principles is desired, the program must stress mathematical meaning and insight into number. Experiences in which pupils use equipment will help build a concept of the meaning of fractions. The following devices may be used advantageously:

1. Fraction kits for learning the relation of fractional parts to a whole, to each other, and for addition and subtraction of fractions and mixed numbers;

2. Measuring cups graduated to show halves, thirds, and quarters, or containers in the shape of rectangular solids which will hold a pint and a quart;

3. Foot rulers graduated in eighths or sixteenths; and

4. Fractioneers, simplified protractors for dividing circles into fractional parts. The latter can be used to advantage in making accurate fractional parts for individual fraction kits for pupils.

The greater the opportunity for concrete experiences in which the pupil manipulates objects to show a fractional amount, the greater the probabily that he will comprehend the meaning of a fraction as a part of a unit.

Decimal fractions could be more easily taught by means of instruments such as micrometers, rulers, clinical thermometers, and automobile odometers, as well as with maps, charts, and surveyor's tapes. The use of circular discs or diagrams divided into tenths also helps children visualize a tenth.

In the upper elementary grades many students have had difficulty understanding why the factor $\frac{1}{2}$ occurs in the formula $A=\frac{1}{2}\,bh$. Similarly, they have forgotten

[4] Emil Berger and Donovan A. Johnson, *A Guide to the Use and Procurement of Teaching Aids for Mathematics.* (Washington, D. C.: National Council of Teachers of Mathematics, 1959), 75¢.

that the volume of a cone contains the factor $\frac{1}{3}$. Why have they failed to retain this information? Possibly we have placed too much emphasis on one means of conveying information or ideas—telling. Place a manipulative aid in the hands of a student, and permit him to examine the pieces illustrating that the area of a triangle is one-half of the parallelogram. In developing the formula for the volume of a cone, hand the student a hollow cone and a hollow cylinder with the same base and altitude. Let him discover the relationship between the two containers by pouring the contents of a cone filled with sand into a cylinder. He will not forget that the formula for the volume of a cone contains the factor $\frac{1}{3}$. The employment of a combination of visual, auditory, tactual, oral and muscular sensations has great power in helping to form new thought patterns. One method of learning is not likely to be as effective as a combination of methods.

If you as an arithmetic teacher have experienced conveying a process or concept in arithmetic to young boys and girls with the use of multi-sensory aids, you are cognizant of the contribution that aids can make towards learning. Now through NDEA funds you have an opportunity to build a mathematics laboratory. It is worth all of the energy that it takes to build and organize. Multisensory aids will help children to visualize arithmetical facts and principles and thus develop the power to discover and understand arithmetic.

The enrichment of the program for advanced pupils is an accepted practice in many schools. It has been observed, however, that in many cases the enrichment for the able pupil is more often found in language arts, reading, social studies, art, and music than in mathematics. One method of enrichment in arithmetic is through reading. The effectiveness of the teaching of arithmetic could be multiplied many times if the

Display of aids for teaching arithmetic acquired by Livingston Elementary School, Covington, Georgia, from NDEA Title III funds.

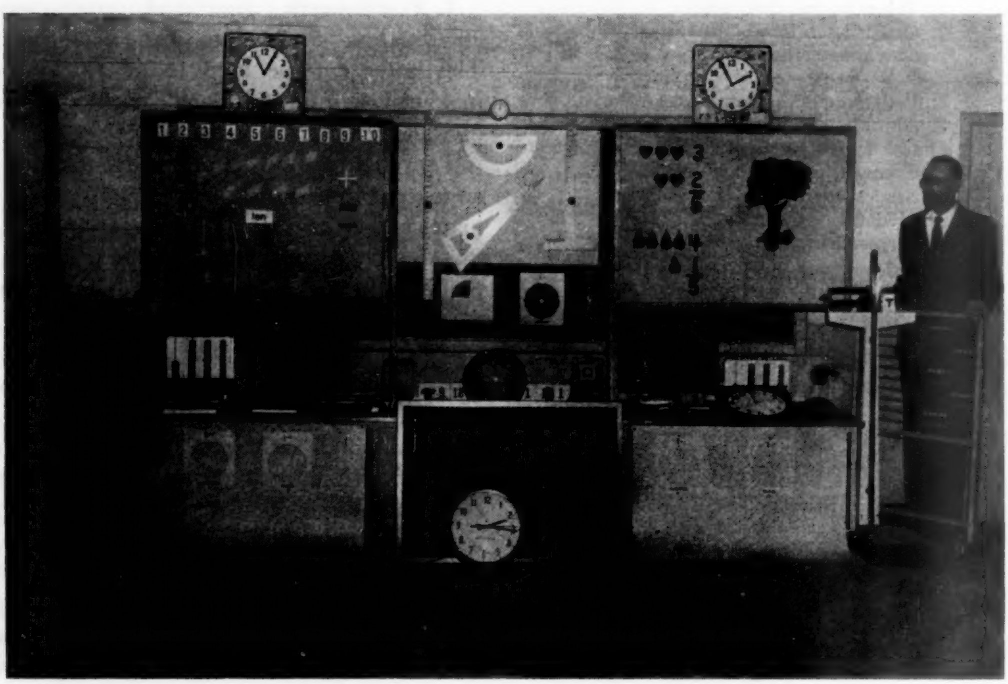


TABLE I

TOTAL FUNDS INVOLVED FOR APPROVED PROJECTS IN FISCAL YEAR 1959 UNDER TITLE III OF NDEA

49 States and Territories	No. of Projects[1]	Amount	Per cent
GRAND TOTAL	10,570[2]	$23,544,713	100.0
Science	6,286	18,035,903	76.6
Mathematics	2,601	1,761,923	7.5
Language	1,683	3,746,887	15.9
GRAND TOTAL	10,570	23,544,713	100.0
Elementary	2,971	3,942,444	16.8
Secondary	5,520	13,332,942	56.6
Combined Elem. & Sec.	2,079	6,269,326	26.6
Science Total	6,286	18,035,903	100.0
Elementary	1,945	3,210,958	17.8
Secondary	3,044	9,626,846	53.4
Combined Elem. & Sec.	1,297	5,198,099	28.8
Mathematics Total	2,601	1,761,923	100.0
Elementary	783	422,647	24.0
Secondary	1,330	813,361	46.2
Combined Elem. & Sec.	488	525,914	29.8
Language Total	1,683	3,746,887	100.0
Elementary	243	308,839	8.2
Secondary	1,146	2,892,735	77.2
Combined Elem. & Sec.	294	545,313	14.6

[1] The project unit may be a district, township, county, or combination thereof.

[2] When a project included more than one field, it was counted once for each field represented. Thus, a project including science, mathematics, and modern foreign language counted as three separate projects in this Table.

NOTE: All money rounded to nearest dollar; therefore, detail may not add to total.

school library contained a sufficient number of mathematics books. The purchase of such reference books other than textbooks, for elementary pupils, is eligible for reimbursement under Title III. The guide[5] written by Berger and Johnson has an excellent list of library books for primary grades, lower elementary grades, upper elementary grades, and grades 7–9. A teacher does not need to wait for special administrative arrangement of students to challenge the able student. Every teacher can do a better job of enrichment in mathematics for all students through a better equipped library. The library can be the nucleus of such a program.

[5] Berger and Johnson, *op. cit.*

Many teachers feel that it is the job of the administrator to see that the teacher has the necessary supplies and equipment. This may be true to some extent, but the teacher of arithmetic should certainly inform the administration of her needs. The mathematics teacher should be aware that teachers in other subject areas at both the elementary and secondary levels are seeking a share of each school dollar. A break-down of the money spent under Title III shows that many more dollars are being spent for science and modern foreign languages than for mathematics.

Table I above classifies in certain categories the funds involved for approved projects in science, mathematics and modern foreign languages for elementary and sec-

ondary levels under Title III. It should be understood that where "combination" appears in columns of Table I, reference is made to a type of administrative unit which originated the projects. Such units include both elementary and secondary grade levels and projects in such administrative units might actually involve either elementary or secondary grades or both.

Significantly, there were more projects involving mathematics than projects involving modern foreign languages. However, a larger share of the total funds was approved for projects involving modern foreign languages than for projects involving mathematics. The fact that equipment for modern foreign language laboratories is more elaborate and costly probably explains this difference.

Of the total funds approved for acquisition of equipment and minor remodeling, 76.6%, 7.5% and 15.9% were for science, mathematics and modern foreign languages respectively. Your attention is called to the fact that approximately ten times more money is being spent for acquisition of equipment and minor remodeling in science than in mathematics, and more than twice as much for modern foreign languages as for mathematics.

Table II shows that the per cent of funds allocated to approved elementary projects according to subject matter fields was 17.8%, 24.0% and 8.2% for science, mathematics, and modern foreign languages respectively. It should be emphasized that the percentages represented in Tables II and III indicate relative weights given the elementary and secondary school projects within each field and have no relation to the total dollar expenditures made in the separate fields.

In the elementary schools, more money is being spent percentagewise for acquisition of equipment and minor remodeling in mathematics than for science or modern foreign languages. More dollars are being spent for science than for mathematics in the elementary schools but more money is being spent for mathematics than modern foreign

TABLE II

PER CENT OF FUNDS EXPENDED IN EACH SEPARATE FIELD DEVOTED TO ELEMENTARY SCHOOL PROJECTS

Administrative Units	Science	Mathematics	Modern Foreign Language
Separate elementary units	17.8%	24.0%	8.2%
Elementary portion from combined elementary and secondary units (estimate)	7.2	10.2	1.4
Total for elementary	25.0	34.2	9.6

TABLE III

PER CENT OF FUNDS EXPENDED IN EACH SEPARATE FIELD DEVOTED TO SECONDARY SCHOOL PROJECTS

Administrative Units	Science	Mathematics	Modern Foreign Language
Separate secondary units	53.4%	46.2%	77.2%
Secondary portion from combined elementary and secondary units (estimate)	21.6	19.6	13.2
Total for secondary	75.0	65.8	90.4

languages. More children are required to study mathematics than either science or the foreign languages.

Let us assume that the funds involved for combined elementary and secondary administrative units found in Table I are divided in the same proportions as in the separate elementary and secondary administrative units. In this assumption, the elementary portions from the combined elementary and secondary units are 7.2%, 10.2%, and 1.4% for science, mathematics and foreign languages respectively. Table II indicates that, of the total amounts spent for the acquisition of equipment and minor remodeling under Title III, the elementary schools received in the fiscal year 1958–59 twenty-five cents of each dollar spent in science, thirty-four and two-tenths cents of each dollar spent in mathematics, and nine and six-tenths cents of each dollar spent in modern foreign languages.

The per cent of funds allocated to approved secondary projects according to subject matter fields is found in Table III. It is easily seen that the secondary schools are getting the lion's share.

Instruction in arithmetic can be appreciably improved through the purchase and use of teaching aids. Every dollar should be spent wisely. Co-operate with your local administrators and your state department of education to get all possible support to strengthen your mathematics program.

Editor's Note. The disproportionate amount of Defense Act money allotted to elementary mathematics does not need to alarm us if we are sure our teachers are equipped with the devices they will use. Admittedly, arithmetic doesn't take as much equipment as science. The right proportions are difficult to assign.

We will need to watch that such equipment as is purchased is used. It is fairly common experience to find devices of the kind mentioned gathering dust, or with parts missing.

The recognition of the place of these devices is a recent victory; the continuing and optimum use of them is still to be realized and will require inspired teaching in our colleges and wise supervision in the field.

Christmas Meeting

Plan to attend the Nineteenth Christmas meeting of The National Council of Teachers of Mathematics to be held at Arizona State College, Tempe, Arizona, December 27–30, 1960.

From the Editor—

E. Glenadine Gibb
Iowa State Teachers College, Cedar Falls, Iowa

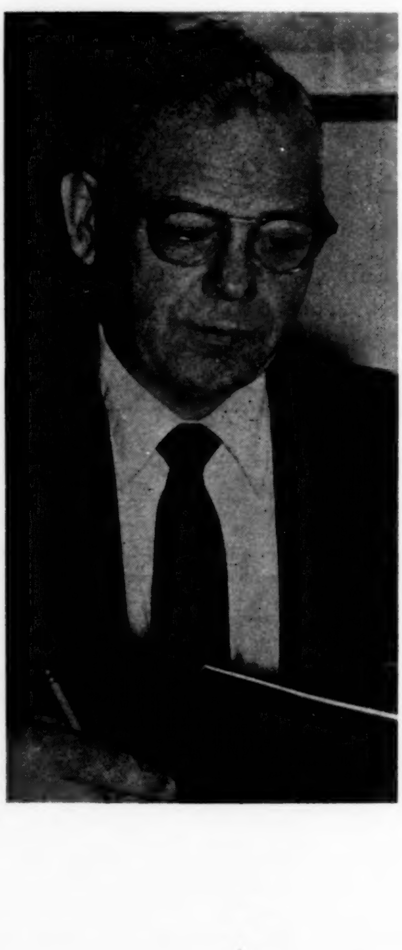
To our first editor, Ben A. Sueltz—On behalf of the past and present editorial staff of The Arithmetic Teacher and its many readers, we wish to thank you for the contribution you have made to mathematics education in your seven years of untiring service as our first editor.

We realize the tremendous responsibility, the hard work, and the creative effort that must be undertaken when starting a new publication. Seven years ago there was no publication devoted to elementary school mathematics. You have made this dream of members of the National Council of Teachers of Mathematics for a special journal a realization.

Some 13,000 readers have come to look forward to receiving their copies of The Arithmetic Teacher. It was you who made it possible for them to keep informed in areas of philosophy, curriculum, research, new ideas for the classroom, new books and materials as they became available, and new programs of improvement. In particular, many readers have expressed gratitude for the personal touch you always brought to each new issue through your Editor's Notes.

Thanks so much, Dr. Sueltz, for all of this. It is indeed far from easy to be your successor!

However, the new editor will make every effort to continue the development of The Arithmetic Teacher in the spirit in which it was created. With the help of her able assistants and the many people interested in elementary school mathematics who will write articles, give suggestions to the editor, and referee manuscripts, it may be possible for her to meet her responsibility for The Arithmetic Teacher and its continued contribution to the teaching of elementary school mathematics.



Dr. Ben A. Sueltz, State University Teachers College, Cortland, New York, first editor of The Arithmetic Teacher.

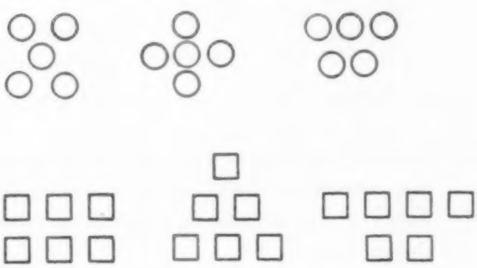
In the Classroom

Edited by Edwina Deans
Arlington Public Schools, Arlington, Virginia

Working with Groups in the Number Span from One Through Nine

A study of groups or patterned arrangements for the numbers through nine helps children build meaning for these numbers and gives them beginning ideas of addition, subtraction, multiplication, and division. The following experiences illustrate some of the many possibilities:

1. Provide each child with nine one-inch cubes or round discs. Have children arrange groups to represent each of the numbers four through nine. Encourage at least three regular, compact pattern arrangements for each number.



2. Transfer grouping arrangements to cards or charts for future use.

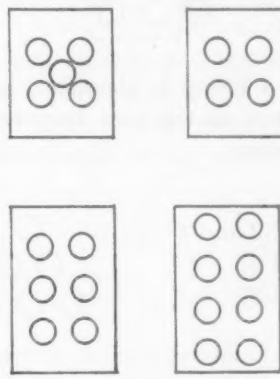
3. Change this group:

Make it one less. Read the new group.
Make it two less. Read the new group.

4. Change this group:

Make it one larger. Read the new group.
Make it two larger. Read the new group.

5. Compare two groups to determine how much larger or how much smaller one group is than another. Arrange groups so that differences are easily recognized.



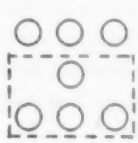
6. Start a group. Find how many more are needed to complete the group:

You have 2.
You want 6.
How many more must you add to make 6?

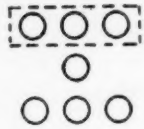
7. Cover a part of a group. See how many are left.

Cover 2. Read the group that is left.

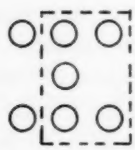
Cover 4. Read the group that is left.



Cover 3. Read the group that is left.

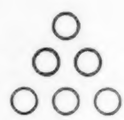


Cover enough so that you will have 2 left. How many did you cover?

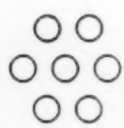


8. After a group is identified, encourage children to tell how they recognized the group.

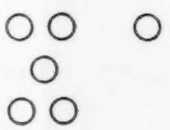
"1 and 2 and 3"
"3 and 3"
"5 and 1"



"2 and 5"
"2 and 3 and 2"
"5 and 2"



9. Build groups for the numbers 6 through 9 using a subgroup of five.



6

7

8

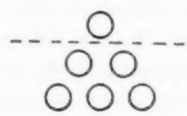
9

Encourage children to read these groups as 5 and 1 or 6; 5 and 2 or 7; 5 and 3 or 8; 5 and 4 or 9.

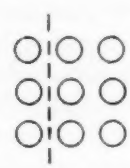
10. Use a marker to separate one group into two smaller groups. Have children read the two parts and identify the total group.

2 at top
4 at bottom
6 in group

3 at left
3 at right
6 in group



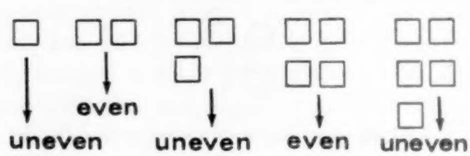
1 at top
5 at bottom
6 in group

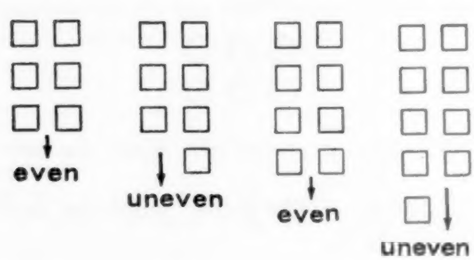


3 at left
6 at right
9 in group

11. Expose a group briefly. See if children can reproduce the group by drawing on paper or by arranging objects to show the pattern.

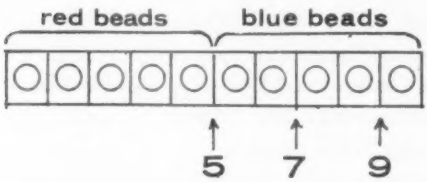
12. Prepare grouping cards showing numbers one through nine arranged in successive patterns of two. Discuss which numbers are even and which are uneven.



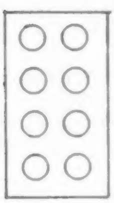


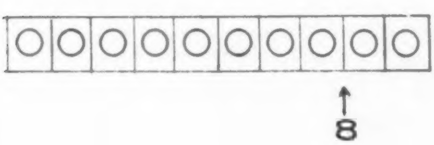
13. Call attention to even and uneven arrangement as children line up by twos for games or activities. If all have partners the group is even. If the last child does not have a partner, the group is uneven.

14. Have each child make a number bead line to 10. Use a strip of one inch squared paper, drawing a large bead in each square. Use one color for the first group of five and another color for the next group of five. Provide a pointed marker for locating numbers according to position along the line.

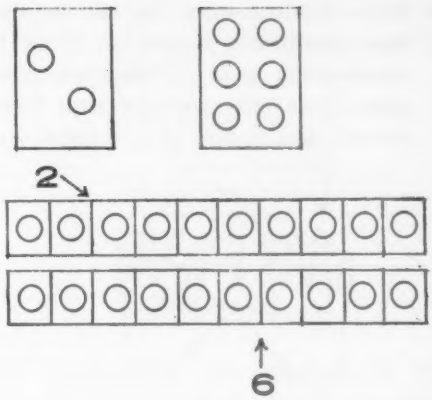


15. Show a group. Have children show where the number represented would come on the bead line.



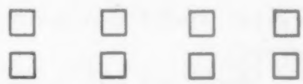


16. Have two children use their bead lines together. Show two groups. Child on the left shows the group on the left; his partner shows group on the right. They see how many match. The ones that do not match tell how many more or how many less one number is than another.

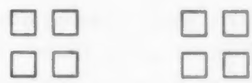


Two beads match. So 2 is 4 less than 6; or 6 is 4 more than 2.

17. Get 8 blocks. Arrange them in groups of 2. How many groups of two do you have?

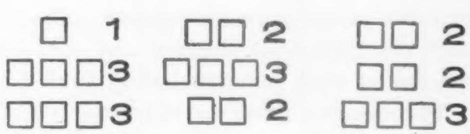


Rearrange them in groups of four. How How many groups of four do you have?



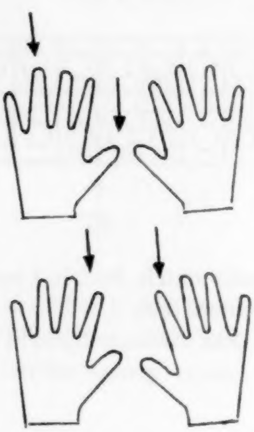
Arrange 6 in groups of 3; how many?
Arrange 6 in groups of 2; how many?
What equal groups can you make with 4 blocks? 9 blocks?

18. Arrange 7 blocks in 3 groups. Have children read the different subgroups they find.



19. Arrange grouping cards in miscellaneous order. Have children arrange them in order from smallest to largest; according to the numbers they represent; from largest to smallest.

20. Have children work together to trace their hands and fingers on 9″ by 12″ construction paper. Place two hands palm down close together with fingers spread. Use fingers as a number line.



Start at left. Find 2; move over 3; where are you?
Find 7. Move back 4 to the left; where are you?
Start at the left. Move over 2; 2 more; 2 more; where are you?

The need to count by ones decreases as children learn position along the line and become adept at using fingers as groups. This type of experience, therefore, is good assurance against the possibility that children will not later use their fingers to count out the answers to facts by ones.

After children have studied several numbers as groups, engaging in experiences similar to the ones described, they are ready to write the symbols for the numbers and to read the number names. The amount of experience with groups which is needed to establish meaning for the numbers will vary with the abilities of children and with the richness of their previous background.

Through experiences with groups you are helping children develop some of the following ideas:

a) Numbers have sequence and order.
b) Each number is made up of all numbers that precede it.
c) A number can be thought of in many ways—as subgroups of a larger group; as more or less than another number; as a part of a larger number.
d) Numbers can be thought of as groups or as proceeding along a line.

Patterns, Relationships, Generalizations

The following ideas and the suggestions for bulletin boards are excerpts from reports prepared by participants in a National Science Foundation-sponsored institute, Mathematics for Elementary School Personnel, which was held at The University of Michigan, Ann Arbor, during the summer of 1959.

The reports from which these illustrations are drawn were prepared by the following: Sister Mary Aletha, Walter E. Bradford, Frances Ceccarini, Grace Cinnamon, Clinton Cook, Jean Doherty, Gladys Grimjes, Mildred Headley, Alice Perkins, Erwin Wong, Willa Sessions.

1. Use patterns and relationships as aids to basic understanding.

Patterns in Multiplication of Seven

$2\times7=14$
$4\times7=28$ (two times as much as 2×7) $2\times(2\times7)$
$8\times7=56$ (two times as much as 4×7) $2\times(4\times7)$
$3\times7=21$
$6\times7=42$ (two times as much as 3×7) $2\times(3\times7)$
$9\times7=63$ (three times as much as 3×7) $3\times(3\times7)$
or
three 7's + six 7's = nine 7's
or
$(3\times7)+(6\times7)=63$
$21+42=63$

Relationships

(A product is unaffected by the order of the factors.)

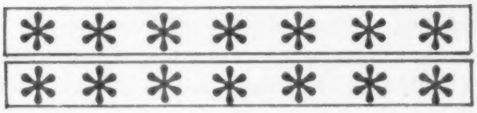
1 group of $7=7$
$1\times7=7$

* * * * * * *

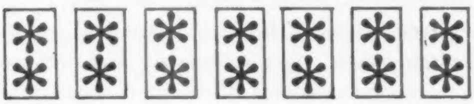
7 groups of $1=7$
$7\times1=7$

* * * * * * *

2 groups of 7 = 14
$2\times 7=14$



7 groups of 2 = 14
$7\times 2=14$



2. Provide some enrichment-type problems through which pupils can observe patterns and begin to make generalizations.

			Cast out nines in product	Cast out fives in product	Cast out sevens in product
$1\times 8=$		8	8	3	1
$2\times 8=$	1	6	7	1	2
$3\times 8=$	2	4	6	4	3
$4\times 8=$	3	2	5	2	4
$5\times 8=$	4	0	4	0	5
$6\times 8=$	4	8	3	3	6
$7\times 8=$	5	6	2	1	0
$8\times 8=$	6	4	1	4	1
$9\times 8=$	7	2	0	2	2
$10\times 8=$	8	0	8	0	3
$11\times 8=$	8	8	7	3	4
$12\times 8=$	9	6	6	1	5

To "cast out" any number from a product, divide the product by that number and record the remainder.

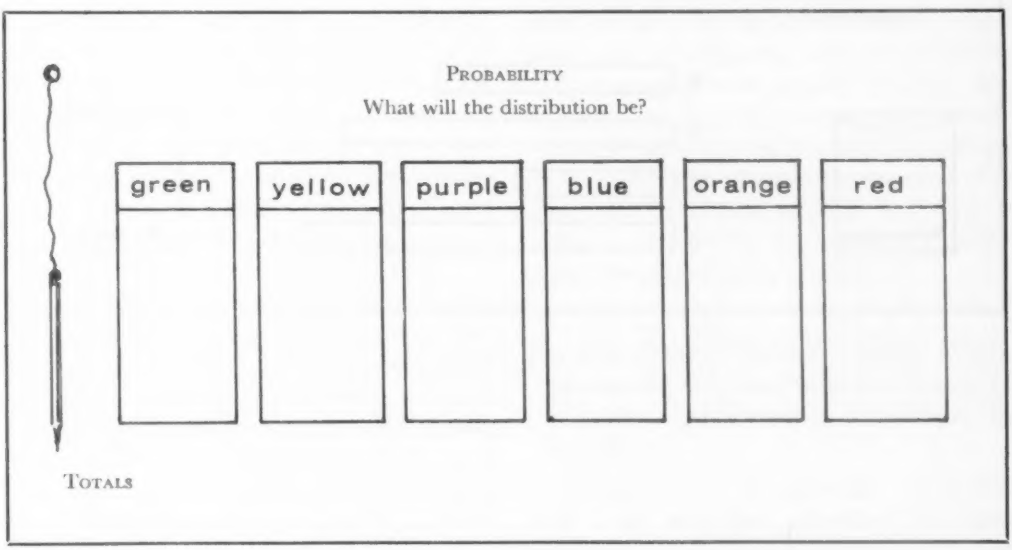
$3\times 1=1$ less than 2×2
$4\times 2=1$ less than 3×3
$5\times 3=1$ less than 4×4
$6\times 4=\_$ less than ___
$7\times 5=\_$ less than ___
$8\times 6=\_$ less than ___
$9\times 7=\_$ less than ___
. . . .
$12\times 10=\_$ less than ___

The product of two consecutive even numbers *or* two consecutive odd numbers is one less than the square of the integer between them.

For Your Bulletin Board

Prepare arithmetic bulletin boards which invite pupil participation.

Take a cube with each side one of the colors shown on the board. Every day for one week each child flips the cube and tallies his or her score on the bulletin board under the proper color. At the end of the week the tallies are totaled and should resolve in a relatively even distribution.

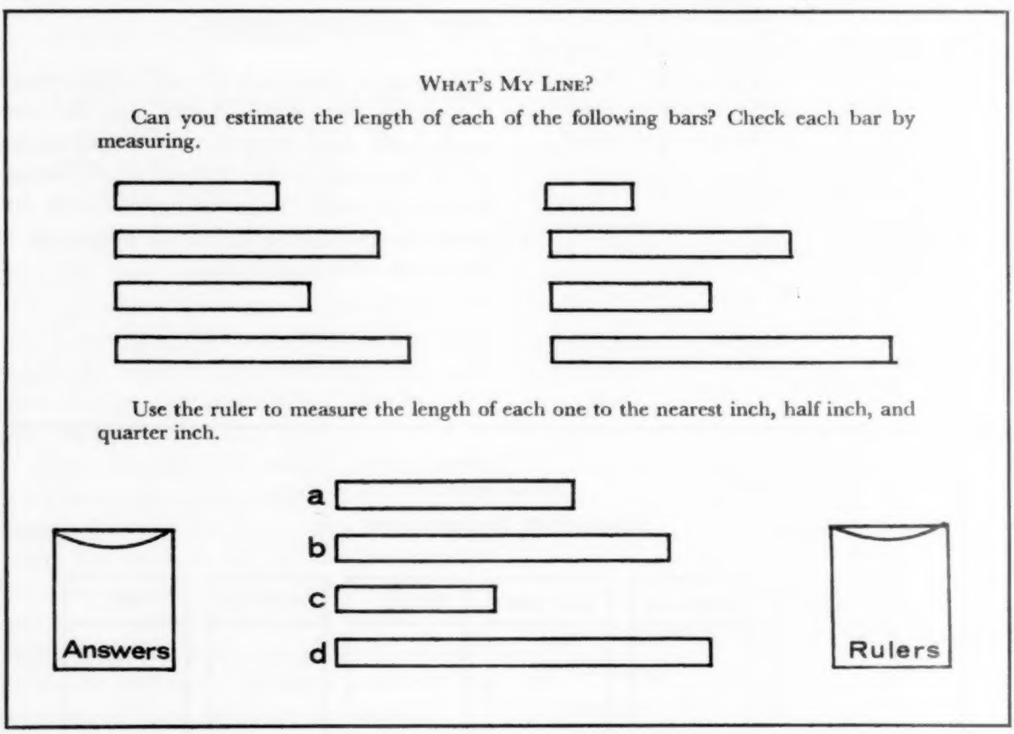


Caution: Cube should have a very even distribution of weight or else one side will be favored. Also keep track of the number of times the cube is thrown. This will probably show that as the cube is thrown more the tallies in each group will be more balanced.

A bulletin board like that shown here could be constructed by the teacher for providing practice in estimating lengths. This activity would serve as a follow-up of the study of measurement. It would also serve as a check for the teacher to discover those children needing further help.

Variations: "What's My Weight? Can you estimate the weight of these articles?" Include various objects such as a book, a letter, a ruler, a box of crayons, and so forth. A scale should be provided so that children could check their estimates.

"What's My Age?" may be used to motivate study on various topics, such as the age of a tree, age of various mathematical terms: decimal point, zero, odometer, and various units of measurement. Through independent study and the use of reference materials, children could gather the information and represent it in the form of a time line.



Reviews of Books and Materials

Edited by CLARENCE ETHEL HARDGROVE
Northern Illinois University, DeKalb, Illinois

Arithmetic for College Students, Edwin I. Stein. Boston: Allyn and Bacon, Inc., 1957. Cloth, xiv+491 pp., $4.50.

The book here under review announces itself as "a comprehensive basic textbook in arithmetic." Part I covers the usual topics of the elementary school curriculum: integers, common fractions, decimal fractions, percentage, and squares and square roots. A small amount of problem material is included.

Part II deals with measures and measurement. A considerable amount of problem material is included in this section, as well as a little material from the area of informal geometry. Many opportunities are afforded for application of skills to concrete situations.

Part III, entitled "Everyday Problems," presents about twenty pages of problem material from the topics usually found in grades seven and eight. Part IV consists of a brief introduction to formulas.

Typically, a new topic, e.g., addition of fractions, is introduced by a "procedure," which is about the same as the "rule" which was found in boldface type in older arithmetic textbooks. This is followed by one or more "sample solutions" for study. After studying these the student takes a diagnostic test which includes examples of each of the types into which the skill has been analyzed. The drill material which follows is keyed to the items in the diagnostic test in such a way as to enable the student to concentrate his practice on those aspects of the skill which most need it. Keying the drill material to the test provides a useful device, if we assume that arithmetic competence is thus atomistically constituted. The drill is more than adequate in amount for students at this level.

This is essentially a drill book. It contains very little instructional material to supplement the teacher's efforts. Little effort is made to supply the rationalizations of the processes. Carrying in addition, "borrowing" in subtraction, why partial products are set down the way they are, why we "invert and multiply" are not explained. If it is assumed that the students who are to use the book already understand the rationale of the number system and the logic underlying the arithmetical processes, and simply need "practice," this book should do the job.

We may assume that readers of this journal are interested in the book under review chiefly from the point of view of its suitability for use in a college course in arithmetic for prospective elementary teachers. Various recommendations for such a course have appeared in the literature. Generally they call for the inclusion of material which will deepen the student's understanding of the processes of arithmetic, broaden his concept of the number system, and extend his appreciation of the logic of the subject—in short, enable him to use and teach arithmetic more intelligently, and not simply to work examples more rapidly or more accurately. This book does not do these things.

There may very well be a place in college mathematics departments for the kind of a course this textbook implies, but such a course would not be the most suitable for prospective elementary teachers.

H. E. BENZ

Five Little Stories, William W. Strader. Washington, D. C.: The National Council of Teachers of Mathematics, 1960. Paper, 16 pp, $0.50.

The stories in this pamphlet are stories only in the sense that accounts of historical events and man's struggle to develop means

adequate to express his mathematical ideas may be called stories. In the absence of a preface or foreword one cannot be certain of the author's intent, but the implication in the title, the drawing on the cover, and much of the phraseology is that these essays are supplementary materials for junior high school students. Yet some ideas expressed both verbally and in mathematical symbolism are too sophisticated to be appreciated by students of this age.

In the reviewer's opinion, the titles of the individual stories do not indicate in every case the nature of the discussion. For example, the first is entitled "An Unbelievable Month of September," but it presents information about the need for calendar reform which had developed and the difficulties encountered when the Gregorian Calendar was adopted. The idea that September, 1752, in England was "unbelievable" seems secondary in this account to the idea that improvement was achieved.

The second story, "Napier's Bones," in addition to presenting some biographical information about Napier, is a relatively straightforward discussion of the relationship of the rods to the gelosia method of multiplication, while the third story traces the origin of the use of the symbol "x" in algebraic expression and comments upon the persistence of this practice. Other historical and current symbols used to represent general or "unknown" numbers are also discussed briefly.

The number discussed under the title "A Colossal, Enormous, Stupendous Number" is $9^{(9^9)}$. By analogy with three in place of nine, the author shows that $9^{(9^9)}$ is very much larger than $(9^9)^9$. By means of logarithms the number of digits needed to express this number in ordinary place-value notation is calculated. The number of volumes similar to those of the *Encyclopaedia Britannica* needed to print the number in this form is also calculated.

In "The Strange Reciprocal of Seventeen" more space is devoted to the decimal expression for the reciprocal of seven than to the reciprocal of seventeen. The cyclic nature of these expressions is the theme.

Each of these stories should pique the curiosity of both student and teacher sufficiently to investigate the topics further among the suggested readings in the bibliographies at the close of each story.

HERBERT F. MILLER

Books Received

The Child's Conception of Geometry, Jean Piaget, Barbel Inhelder, Alina Szeminska (translated from the French by E. A. Lunzer). New York: Basic Books, Inc., 1960. Cloth, vii+411 pp., $7.50.

Geometry for Juniors, Books 1, 2, 3, 4, and Teacher's Book, Grace A. Moss. Oxford: Basil Blackwell and Mott Ltd., 1960. Paper, Books 1, 2, 3, 4, 35 pp. each, 3/6 each; Teacher's Book, 10 pp., 1/-.

Happy Times With Numbers, First Grade, Second Grade, Evelyn Fershing. Boston: Allyn and Bacon, Inc., 1960. Paper, 126 pp.

The Modern Aspect of Mathematics, Lucienne Felix (translated by Julius H. and Fancille H. Hlavaty). New York: Basic Books, Inc., 1960. Cloth, xiii+194 pp., $5.00.

Twenty-fifth Yearbook—Instruction in Arithmetic. Washington, D. C. The National Council of Teachers of Mathematics, 1960. Cloth, viii+366 pp., $4.50 ($3.50 to Council members).

Improvement Projects Related to Elementary School Mathematics

A Selected Listing

Compiled and Edited by J. Fred Weaver
Boston University School of Education, Boston, Massachusetts

"Where can I get information about the nature of ongoing projects related to elementary school mathematics? Can I get the projects' instructional materials to use with children in my own classroom?"

This compilation is designed to help answer questions such as these which are asked many times each day by many different persons.

No attempt has been made at this time to compile a complete listing of such projects. Rather, this is a *selected* listing. It includes those projects that seem to be attracting the greatest attention and interest at the present time. The decision of whether or not to include a particular project in this listing rested solely with the compiler-editor.

Seven of the eleven projects listed in this compilation either deal exclusively with some or all of the elementary grades (K–6) or include some or all of the elementary grades in the projects' full scope:

Sets and Numbers: An Experimental Arithmetic Program.
Geometry for Primary Grades.
Algebra for Grade Five.
School Mathematics Study Group: Project on Elementary Mathematics.
University of Illinois Arithmetic Project.
Syracuse University "Madison Project."
Greater Cleveland Mathematics Program.

Two of the eleven projects listed in this compilation deal expressly with grades 7 and 8 of the junior high school and thus have clear implications for mathematics programs at the elementary school level:

University of Maryland Mathematics Project (Junior High School).
School Mathematics Study Group: Project on Mathematics for Grades 7 and 8.

The two final projects listed in this compilation actually cover grades 9–12. For present purposes, however, special attention has been directed chiefly to the 9th grade portions of these projects. This more or less completes the picture for grades 7–9, which constitute the junior high school level in many school systems.

University of Illinois Committee on School Mathematics Project.
School Mathematics Study Group: Project on Mathematics for Grades 9 through 12.

The following information, when applicable, is given for each project listed in this compilation:

The official name of the project.
The grade level(s) covered by the project (when not evident from its name).
The name of the project director.
The name of any person other than the project director to whom inquiries regarding the project should be sent.
The address of the project office.
References from the professional literature which give an indication of the nature of the project, etc.
Special brochures related to the nature of the project, etc.
Project materials available upon request, including the price of each publication.

The compiler-editor of this listing wishes to express his appreciation to project directors and staff members for their co-operation and assistance in providing the basic information included on each project.

Sets and Numbers: an Experimental Arithmetic Program

(Primary Grades)

Project Director: Prof. Patrick Suppes.

Send project inquiries to: Prof. Suppes.

Project Office address: Serra House, Stanford University, Stanford, California.

Project brochures: Mimeographed reports of the project are available upon request to the project director.

Available project materials:
Sets and Numbers, Book I ($1.50/copy).
Sets and Numbers, Book II ($1.50/copy).
Sets and Numbers: Applications ($1.50/copy).

NOTE: *Teacher's Manual* (free) accompanies each book. Prices listed are approximate cost of pupil's workbooks.

GEOMETRY FOR PRIMARY GRADES

Project Director: Prof. Newton S. Hawley

Send project inquiries to: Prof. Hawley.

Project Office address: Applied Mathematics and Statistics Laboratory, Stanford University, Stanford, California.

References to project in professional literature:

Hawley, Newton S., and Patrick Suppes, "Geometry in the First Grade," *The American Mathematical Monthly* 66 (June-July 1959): 505–506.

Available project materials: Geometry for Primary Grades, Book I, with accompanying *Teacher's Manual* ($2.00/copy).

ALGEBRA FOR GRADE FIVE

Project Director: Prof. W. W. Sawyer.

Send project inquiries to: Prof. Sawyer.

Project Office address: Box 268, Wesleyan University, Middletown, Connecticut.

References to project in professional literature:

Sawyer, W. W., "Why Is Arithmetic Not the End?" THE ARITHMETIC TEACHER 6 (March 1959): 95–96, 99.

Sawyer, W. W., "Algebra in Grade Five," THE ARITHMETIC TEACHER 7 (January 1960): 25–27.

Also refer to issues of *The Mathematics Student Journal*, edited since November 1958 by Prof. Sawyer, for further indication of the project's "spirit," etc.

Available project materials: Booklets are scheduled to be issued at some time in the future by the Wesleyan University Schools Services.

NOTE: Prof. Sawyer's project is, in his words, 'purely an individual effort," being carried on without financial support from a foundation or the like. In this respect his work differs significantly from other projects listed in this report.

SCHOOL MATHEMATICS STUDY GROUP (SMSG): PROJECT ON ELEMENTARY MATHEMATICS

Project Director: Prof. E. G. Begle.

Send project inquiries to: Dr. John Wagner, Assistant to the Director.

Project Office address: Drawer 2502 A, Yale Station, New Haven, Connecticut.

Project brochures: See the SMSG *Newsletter* No. 4, March 1960 (in particular, pages 9 and 14). Interested persons may write to the Project Office and ask to be placed on the mailing list for the SMSG *Newsletter*, which is issued from time to time and relates to the many facets of SMSG's total program of work.

Available project materials: None to date.

NOTE: This SMSG Project began operations in the spring of 1960 with special attention to the mathematics program in grades 4–6.

NOTE: Other relevant SMSG projects are listed elsewhere in this report.

UNIVERSITY OF ILLINOIS ARITHMETIC PROJECT (Grades K–6)

Project Director: Prof. David A. Page.

Send project inquiries to: Prof. Page.

Project Office address: University of Illinois, 1207 W. Stoughton, Urbana, Illinois.

References to project in professional literature:

Page, David A., "The University of Illinois Arithmetic Project," *The American Mathematical Monthly* 66 (May 1959): 419–422.

Page, David A., "Do Something about Estimation!" *Updating Mathematics*, Section II; Vol. 2, No. 8 (New London, Conn.: Arthur C. Croft Publications, April, 1960).

Page, David A., "Probability," *The Growth of Mathematical Ideas, Grades K–12*, Twenty-Fourth Yearbook, National Council of Teachers of Mathematics (Washington, D. C.: the Council, 1959). Pp. 229–271. In relation to the latter, Prof. Page says: "There is no specific reference to the project in this article but it indicates some of the general philosophy of the project."

Project brochures: "Information Concerning The University of Illinois Arithmetic Project." Interested persons may write to the Project Office and ask to be placed on the mailing list for pertinent Project information and progress reports as released.

Available project materials: A document entitled "An Example from The University of Illinois Arithmetic Project" is available as a means of giving interested persons some indication of the kind of work done in experimental classes associated with the project. Reprints of certain articles are also available. Units on special topics are in preparation.

NOTE: The University of Illinois Arithmetic Project is distinct from, and not a downward extension of, the UICSM Mathematics Project listed elsewhere in this report.

THE SYRACUSE UNIVERSITY "MADISON PROJECT"
(Grades 3–10)

Project Director: Prof. Robert B. Davis.

Send project inquiries to: Prof. Davis. However, anyone wishing to visit project classes at Weston (Conn.), Tarrytown (N. Y.), or Syracuse (N. Y.) should write to: for Weston, Mrs. Beryl Cochran, Weston Public Schools, Weston, Conn.; for Tarrytown, Miss Cynthia Parsons, Tarrytown Public Schools, Tarrytown, N. Y.; for Syracuse, Mr. William Bowin, Board of Education Building, West Genesee St., Syracuse, N. Y. Connecticut teachers also should note a further exception below.

Project Office address: Mathematics Department, Syracuse University, Syracuse 10, New York.

References to project in professional literature:

Davis, Robert B., "The Mathematics Revolution: Causes and Directions," *Frontiers of Secondary Education III*, Chapter 7 (Syracuse, N. Y.: Syracuse University Press, 1958).

Davis, Robert B., "Seventh-Grade Algebra: The Madison Project," *New York State Mathematics Teachers Journal* 9 (1959): 5–8.

Davis, Robert B., "How Do We Learn Mathematics?" *M.I.T. Technology Review* 62 (December 1959): 28–30.

Davis, Robert B., "The Syracuse University 'Madison Project'." *American Mathematical Monthly* 67 (February 1960): 178–180.

Davis, Robert B., "Mathematics for Younger Children—The Present Status of the Madison Project," *New York State Mathematics Teachers Journal* 10 (April 1960): 75–79.

Davis, Robert B., "The 'Madison Project' of Syracuse University. " To be published in the fall of 1960 in *The Mathematics Teacher*.

Davis, Robert B., "The Madison Project: Algebra in Grades 3–10." To be published in the *Grade Teacher*.

Lutz, Marie, "The Madison Project," THE ARITHMETIC TEACHER 6 (December 1959): 320–321.

Parsons, Cynthia, "Algebra in the Fouth Grade," THE ARITHMETIC TEACHER 7 (February 1960): 77–79.

Project brochures: Various free materials are available upon request.

Available project materials:

Madison Project Workbook, with accompanying *Teacher's Manual* ($5.00/set of both).

Tape recordings and filmstrips of project classes available in the fall of 1960; price not yet determined. (These will be available to Connecticut teachers under a special arrangement by writing to Dr. Mary Tuloch, Connecticut State Department of Education, Hartford, Conn. Teachers in all other states should write to the Madison Project office for these materials.)

THE GREATER CLEVELAND MATHEMATICS PROGRAM (G.C.M.P.)
(Grades K–12)

Project Director: Dr. James F. Gray.

Send project inquiries to: Prof. B. H. Gundlach, Chief Consultant of G.C.M.P., Bowling Green State University, Bowling Green, Ohio.

Project Office address: Educational Research Council of Greater Cleveland; 75 Public Square, Cleveland 13, Ohio.

References to project in professional literature:

Baird, George H., "Greater Cleveland Mathematics Plan," *The American Mathematical Monthly* 67 (April 1960): 376.

Project brochures: G.C.M.P. Bulletin, Vol. 1, No. 1, March 11, 1960; Vol. 1, No. 2, March 25, 1960; Vol. 1, No. 3, May 13, 1960.

Available project materials:

ENGAP (Elementary Number Games and Puzzles), for grades 1–12 ($4.00/copy).

Suggested Mathematics Topics for Review and Enrichment, for grades 5–8 ($2.00/copy).

Principal's Notebook, lecture notes presented by Prof. Gundlach to teachers in grades K–6 ($8.00/copy).

Lecture Notes, Sample kit of selected grade level lecture notes ($1.75/kit).

Supplemental Notes—TV Programs, Set of 8 supplemental teacher's notes based on lectures from the TV series, "Tomorrow's Learning Today," covering grades K–9 ($1.25/set).

Tentative Arithmetic Outline, for grades K–6 ($0.25/copy).

Numerals-Relations Kit, Sample kit of plastic numerals and relations used in G.C.M.P. ($0.60/kit).

Teacher's Guides for grades K, 1, 2, and 3 will be available in the fall of 1960; price, about $1.50 each.

Up-to-date price lists of G.C.M.P. materials are released periodically.

NOTE: G.C.M.P. operates under the Educational Research Council of Greater Cleveland, a unique organization representing a co-operative venture between the school and business and industry, whose "major concern is how to use research results and apply them to the problems of

elementary and secondary schools." (See pp. 96–102 of *School Management* for February 1960.)

UNIVERSITY OF MARYLAND MATHEMATICS PROJECT (JUNIOR HIGH SCHOOL) (*UMMaP*)

Project Director: Dr. John R. Mayor.

Send project inquiries to: Mrs. Helen L. Garstens, Associate Director.

Project Office address: College of Education, University of Maryland; College Park, Maryland.

References to project in professional literature:

Garstens, Helen L., Keedy, M. L., and Mayor, John R., "University of Maryland Mathematics Project," THE ARITHMETIC TEACHER 7 (February 1960): 61–65.

Keedy, M. L., "The University of Maryland Mathematics Project," *The American Mathematical Monthly* 66 (January 1959): 58–59.

Project brochures: Progress reports are issued periodically to persons on the project mailing list.

Available project materials:

Mathematics for the Junior High School, Book 1 ($3.00/copy).

Teachers Guide, Book 1 ($2.00/copy).

Mathematics for the Junior High School, Book 2 ($3.00/copy).

Teachers Guide, Book 2 ($2.00/copy).

SCHOOL MATHEMATICS STUDY GROUP (SMSG): PROJECT ON MATHEMATICS FOR GRADES 7 AND 8

Projector Director: Prof. E. G. Begle.

Send project inquiries to: Dr. John Wagner, Assistant to the Director.

Project Office address: Drawer 2502 A, Yale Station, New Haven, Connecticut.

Project brochures: See the SMSG *Newsletter* No. 3, September 1959 (a resume of teacher comments on experimental use of the "Junior High School Mathematics Units"); also *Newsletter* No. 4, March 1960. Interested persons may write to the Project Office and ask to be placed on the mailing list for the SMSG *Newsletter*.

Available project materials:

Junior High School Mathematics Units ($2.75/complete set).

Vol. I, "Number Systems" ($0.75/copy).
Commentary for Teachers for Vol. I ($0.75/copy).

Vol. II, "Geometry" ($0.50/copy).
Commentary for Teachers for Vol. II ($0.50/copy).

Vol. III, "Applications" ($0.25/copy).
Commentary for Teachers for Vol. III ($0.25/copy).

Mathematics for Junior High School

Vol. I; also, *Commentary for Teachers* for Vol. I ($3.50/set of both).

Vol. II; also, *Commentary for Teachers* for Vol. II ($2.00/set of both).

NOTE. Other relevant SMSG projects are listed elsewhere in this report.

UNIVERSITY OF ILLINOIS COMMITTEE ON SCHOOL MATHEMATICS (UICSM) PROJECT (Grades 9–12)[1]

Project Director: Prof. Max Beberman.

Send project inquiries to: Prof. Beberman.

Project Office address: University of Illinois, 1208 W. Springfield, Urbana, Illinois.

References to project in professional literature:

UICSM Staff, "The University of Illinois Mathematics Program," *The School Review* 65 (Winter 1957): 457–465.

UICSM Staff, "Words, 'Words,' "Words"," *The Mathematics Teacher* 50 (March 1957): 194–198.

UICSM Staff, "Arithmetic with Frames," THE ARITHMETIC TEACHER 4 (April 1957): 119–124.

(Other references are included on the "Information Sheet" mentioned below.)

Project brochures:

"Information Sheet" (February 1960).

"Illustrations of Content from Units 1, 2, 3, and 4."

Available project materials (order from the University of Illinois Press, Urbana):

High School Mathematics:

Unit 1, "The Arithmetic of Real Numbers."

Unit 2, "Pronumerals, Generalizations, and Algebraic Manipulation."

Unit 3, "Equations and Inequations; Applications."

Unit 4, "Ordered Pairs and Graph."

(Units 1–4 combined: Student's Edition, $2.25/copy).

(Units 1–4 combined: Teacher's Edition, including Student's Edition, $6.00/copy).

Unit 5, "Relations and Functions" (Student's Edition, $1.50/copy; Teacher's Edition, $3.00/copy).

Unit 6, "Geometry" (Student's Edition, $1.50/copy; Teacher's Edition, $3.00/copy).

Other Units are in use experimentally or are in preparation.

[1] In some schools the UICSM program is begun in Grade 8.

Note: The UICSM Project is distinct from the University of Illinois Arithmetic Project listed elsewhere in this report.

School Mathematics Study Group (SMSG):

Project on Mathematics for Grades 9–12

Project Director: Prof. E. G. Begle.

Send project inquiries to: Dr. John Wagner, Assistant to the Director.

Project Office address: Drawer 2502 A, Yale Station, New Haven, Connecticut.

Project brochures: See the SMSG *Newsletter* No. 4, March 1960, which includes a statement of SMSG's objectives, procedures, progress, and announcement of availability of publications as of May 1960. Interested persons may write to the Project Office and ask to be placed on the mailing list for the SMSG *Newsletter*.

Available project materials:

Mathematics for High School:

First Course in Algebra, with *Commentary for Teachers* ($4.25/set of both).

(The above materials are for Grade 9. Materials available for Grades 10–12 are beyond the scope of this report and hence are not included here.)

Note: Other relevant SMSG projects are listed elsewhere in this report.

Looking to the Future

The editors of The Arithmetic Teacher are most anxious to keep readers informed of ongoing improvement projects relating to elementary school mathematics. The foregoing compilation is the first step in a systematic attempt to accomplish this.

Information on these and other projects will appear in one form or another in various future issues of The Arithmetic Teacher. We invite persons and groups working on significant experimental projects dealing with the improvement of elementary school mathematics programs to communicate with Professor Weaver[2] so that, whenever possible, we may disseminate appropriate information about the projects in The Arithmetic Teacher.

[2] Boston University School of Education, 332 Bay State Road, Boston 15, Mass.

The National Council of Teachers of Mathematics

Minutes of the Annual Business Meeting

Statler-Hilton Hotel
Buffalo, New York
April 22, 1960

Dr. Harold P. Fawcett, President, called the meeting to order at 4:00 P.M.

I. A motion was made, seconded, and passed to approve the minutes of the meeting of April 3, 1959, as printed in the *Journals*.

II. *Report of the Recording Secretary.*

The Board of Directors has held three meetings since the last meeting of the Council in Dallas in 1959. These meetings were as follows: Ann Arbor, Michigan, August 16, 1959; Washington, D. C., December 18–19, 1959; and Buffalo, New York, April 19–20, 1960. In the brief time allowed, it is impossible to relate all of the actions of the Board. Much of the work is routine business, receiving reports from a large number of committees, and in discussing at length the work of the Council. This report merely attempts to inform the membership of the major actions taken.

Ann Arbor Meeting

A. A decision was made at the Dallas meeting to add two new staff members to the Washington Office. These were: a professional assistant and an editorial assistant. The persons selected for these new positions were formally announced at the Ann Arbor meeting. They are: Mr. James Brown, professional assistant, and Miss Miriam Goldman, editorial assistant. The addition of these two new staff members has proved to be of great aid in the work of the Council.

B. Dr. E. Glenadine Gibb, of Iowa State Teachers College, was appointed to the position of Editor of THE ARITHMETIC TEACHER. She succeeds Dr. Ben Sueltz, the first editor, who has done a fine job in developing this outstanding journal. Dr. Sueltz has served the maximum time allowable.

C. Final plans for the Policy Conference were adopted. This Conference will be referred to later.

D. The Board approved submission of a proposal to the National Science Foundation for a grant to sponsor regional meetings with supervisors and administrators on a survey of the present curriculum studies. This proposal has been granted by the National Science Foundation. The program is under the directorship of Frank B. Allen. During the fall of 1960 there are to be eight regional conferences in various parts of the country.

Washington Meeting

At the Dallas meeting, the Board adopted the recommendation of the Problem-Policy Committee to hold a Policy Conference in October, 1959. A detailed report of this action is to be found in the minutes of the meeting of the Council as published in the October, 1959, issues of *The Mathematics Teacher* and THE ARITHMETIC TEACHER. As a result of this Conference, the Board was called into session on December 18, 1959, in Washington. Most of the work of this meeting centered around the recommendations of the Policy Conference.

A. New Committee Structure:

The standing committees of the National Council of Teachers of Mathematics shall consist of the following:

1. An Executive Committee
2. A Committee on Professional Standards and Status
3. A Committee on Professional Relations
4. A Committee on Publications
5. A Committee on Educational Policies and Programming

The Executive Committee shall consist of the president and four members of the Board of Directors selected for one year by the president and approved by the Board. The other four standing committees shall have five members each, appointed for a three-year

term by the president with the approval of the Board; except that the initial terms shall be one year for one member, two years for two members, and three years for two members. A committee member may serve no more than two successive terms.

Each committee shall be responsible to the Board of Directors but may appoint such subcommittees as deemed necessary to accomplish the responsibilities delegated to it.

Until the Bylaws are revised, the Executive Committee shall appoint a subcommittee on budget and auditing and on nominations and elections. The Publications Committee shall appoint a subcommittee on yearbook planning and one on supplementary publications. Also the Executive Committee shall, until the Bylaws are revised, consist of the president and two members of the Board.

The president shall act as chairman of the Executive Committee. The president shall each year select from each of the other committees one member who will serve as chairman.

This organization shall become effective immediately following the annual meeting in 1960 and all committees and representatives now in existence shall be dismissed or incorporated in the new structure.

Each committee is free to establish its method of operation and its policies except those specifically denied it by the Board.

B. Elementary School Mathematics Study:
One of the strongest recommendations of the Policy Conference was to the effect that the Council formulate a plan for a complete study of elementary mathematics. The Board adopted this recommendation and authorized the Executive Committee and the Chairman of the Committee on Educational Policies and Programming to develop the procedure and get the work started.

C. Information-Evaluation Study:
In view of the many different projects extant in the field of curriculum matters and related topics, the Board authorized that a plan be devised whereby information and an evaluation of these projects might be made available to the membership of the Council and to the public concerned. The procedure for this study is to be developed by the Executive Committee and the Chairman of the Committee on Educational Policies and Programming.

D. The Board agreed to co-sponsor with the U. S. Department of Education a conference on In-service Reeducation of Mathematics Teachers. This Conference was held in Washington on March 18, 19, and 20 under the direction of Dr. Kenneth Brown and Mr. Myrl Ahrendt.

Buffalo Meeting

A. Budget:
It is well known that the Council is one of the most rapidly growing educational organizations. As the Council grows, both income and expenditures naturally increase. The Board adopted a budget of $251,225 for the year 1960–61. Since this budget is being published in the October issues of *The Mathematics Teacher* and The Arithmetic Teacher, details will not be given here.

B. Bylaws:
For a number of reasons, the Board decided the Bylaws should be revised. A committee was appointed to study this matter. The Board approved the report of the Committee. Since the proposed revisions must be made available to the membership at least thirty days before a meeting of the Council where changes are to be made in the Bylaws, the suggested changes will not be read at this time.

C. The Board has felt for some time that the Council must provide a system of retirement income, for the paid staff, to be in harmony with other organizations. In view with its thinking the Board has agreed to adopt a plan as provided by T.I.A.A.

D. Since the work in the Washington Office has grown to such large proportions, the Board has decided to employ the services of a business management company to make a survey of our present operations and to submit recommendations for improvements should such be warranted.

E. Future Conventions:
Approved to date

1960 Summer: Salt Lake City, Utah
Christmas: Tempe, Arizona
1961 Spring: Chicago, Illinois
Summer: Toronto, Canada
1962 Spring: San Francisco, California
Summer: Madison, Wisconsin
1963 Spring: Pittsburgh, Pennsylvania
Summer: Eugene, Oregon
1964 Spring: Miami Area, Florida

III. *Report of the President.*

President Fawcett gave the following report:

The report to which you have just listened, summarizing important actions of the Board of Directors during the past year, reflects the increasing activity of the National Council in its continuing effort to improve the teaching of mathematics. Perhaps the most significant event during this period was the Chicago Policy Conference, recommended by the Problem-Policy Committee. You have undoubtedly read the preliminary report of this conference published in the November, 1959, issue of *The Mathematics Teacher*, and the complete report will appear in the April, 1960, issue. To translate into well-considered action the policies proposed at this conference is one of the major tasks now facing the Board of Directors.

During the year the administrative machinery of the Council has squeaked along more or less satisfactorily. There has scarcely been a day when I have not been grateful to the Board for its wisdom in creating at Dallas a program committee to relieve the president of major responsibility for planning and arranging the program of the annual meeting. The Board is indebted to this committee for the excellence of the program now in process and especially to Glenadine Gibb and Ida May Puett who served as co-chairmen of the committee. To the gratitude of the Board, I wish to add my own personal note of thanks.

During the year, 26 other committees serving the basic purposes of the Council have been at work in varying degrees. To the 142 men and women who have served as members of these committees, and especially to the respective chairmen, I wish to express my thanks and appreciation. As you will note in the April issue of *The Mathematics Teacher*, the Board has reorganized the committee structure of the Council, better to serve its expanding interests, and the role of the present committees within the new structure is yet to be determined.

Representatives of the National Council sit on boards, commissions and committees of other professional organizations and to the eight men and women now serving in this capacity we are indeed grateful.

The Council is to be congratulated on the quality of its three *Journals*. Grateful readers from time to time testify to the excellence of these publications and the respective editors are to be highly commended for the quality of their services. It is appropriate that Dr. Ben Sueltz be specifically mentioned since he is now retiring as editor of THE ARITHMETIC TEACHER. Six years ago, at the request of the Board, he launched this publication, and its steady growth is silent testimony to his creative genius. As its only editor, he has now completed the full complement of time allowed under our Bylaws. The new editor is Dr. Glenadine Gibb, and our best wishes are with her as she adds this additional responsibility to her busy professional life.

Included in the budget for the past year was an item to provide a half-time secretary for the president. This increased secretarial service, along with relief from program planning, made it possible to meet the minimum obligations of the president's office, but in my judgment that is not enough, for to be satisfied with the minimum is to approve a static position which is certainly untenable. We expect the president to keep the machinery running, but we also expect him to provide a quality of creative leadership which will indeed be instrumental in "promoting the interests of mathematics in America, especially in the elementary and secondary fields." The opportunities for service are almost unlimited, and he could, in fact, easily invest his full time in professional activities directly related to the president's office. In the usual situation, however, he is carrying a full load of responsibility in the institution with which he is associated, and under such circumstances one might wonder whether the best interests of the Council are likely to be served, especially the leadership function. Faced with pressures which make demands beyond human strength, man is likely to neglect those obligations which speak most softly. Is the time spent in class preparation really necessary? Why all this study? Go to class unprepared. Who is there to know the difference? And so through my own limitations, an organization dedicated to the improvement of mathematics education is indirectly responsible for lowering the quality of classroom performance. Even so, I could do little better than keep the machinery running, and, while I am quite willing to concede my own inefficiencies, it is my considered judgment that there is a problem associated with this office which calls for thoughtful study. With a half-time secretary the clerical and stenographic needs of the office *can* be met. Perhaps a full-time secretary with a flair for administrative responsibility could assist the president in keeping the machinery running and thus release him for more productive leadership. To reimburse the institution with which he is associated for half his time is another possibility which seems to have considerable merit.

The major responsibility of The National Council of Teachers of Mathematics is to provide its membership with a quality of service which will enrich the teaching of mathematics in the classrooms of America. It is that purpose which gives direction and emphasis to the programs of our national meetings, and it is to that end that our publications program is directed. I wish now to propose that we increase our service to Council members through providing them from time to time without cost such supplementary publications as selected monographs of general value, reprints of important articles on mathematics education, and the like. The distribution of "The New World of Mathematics" approximately one year ago has undoubtedly stirred many mathematics teachers into a program of professional improvement, and students in many classrooms are for the first time *seeing* mathematics with both physical and intellectual eyes because their teachers received a copy of "A Guide to the Use and Procurement of Teaching Aids for Mathematics" prepared by Emil Berger and Donovan Johnson. To provide such service is to build good will for the Council, to nourish an appetite for comparable publications, and to help some teachers take

the first long step in a professional program of self-improvement.

I am impressed by the fact that last year our membership increased by 22%, and I am further impressed by the fact that in the official year now drawing to a close approximately 8,000 new names have been added to our membership rolls, an increase of about 31%. But why is that a source of rejoicing? Is it because more pages will now be needed to print our Membership Directory? Is it because our treasury has been increased by close to $40,000? Is size a criterion of value? These outcomes are but vain and empty symbols. We rejoice because the addition of 8,000 new names to the membership rolls means that at least one of our journals is going into homes of 8,000 teachers who never before were under the direct influence of the Council. It means that 8,000 teachers have taken the initial step on the road to professional improvement, and it means that 8,000 good teachers will undoubtedly become 8,000 better teachers. The growth of our membership means that the influence of the Council reaches into an increasing number of mathematics classrooms; the final test of our services is the extent to which the Council improves teaching competence. Let us make it rich and productive.

When I was a small boy in a Canadian school I one day drew a picture of a horse. Since no one knew what it symbolized, I felt compelled to label it, but that in no way made it look any more like a horse. I note from the program that we are now participating in the "Annual Business Meeting" of the National Council of Teachers of Mathematics, but without that label I wonder if it would be recognized as such. Crowded into one hour at the close of a busy day are four scheduled reports and an unknown number of Special Committee Reports. Following a consideration of these $4+n$ reports the remaining time is devoted to "resolutions" and "New Business." Is there anyone in the audience who can suggest a more unpopular time for the introduction of "new business"? Is it likely that under these circumstances, colored by the attractive shadow of an impending banquet, even the "old business" will receive the thoughtful consideration it merits? What are the purposes of this "Annual Business Meeting"? Whatever they are, I cannot believe that they are well served through present procedures which attract such a thin attendance. My own thoughts concerning this problem have yielded no satisfactory solution. It is indeed time for a change.

IV. *Report of the Executive Secretary.*

Mr. M. H. Ahrendt, Executive Secretary, gave the following report.

A. Washington Office:
The full staff of the Washington office during this school year has numbered 18 persons. Two professional staff persons were added with the approval of the Board of Directors: J. James Brown, professional assistant, and Miriam Goldman, editorial assistant. A new staff position of administrative assistant was created to help with correspondence related to inquiries and complaints of a business nature. The establishment of this position has enabled us to give much better service to the persons who complain about nondelivery of their materials by the post office and occasional errors made by our office staff. The increase in membership and our growing sales program required the employment of several additional clerical workers.

The office again outgrew its allotted space, and we were forced to move part of our staff to a room on another floor of the NEA Building. The secondary office houses our Billing and Order Section. An intercom system was added to our office telephones to make it easy for the two parts of the office to keep in touch with each other.

The understanding attitude of the Board of Directors in making available some money for the purchase of office equipment has been much appreciated by the entire staff. We have been able to purchase during the year a postal mailing machine, an improved postal mailing scale, a photocopy machine, and several electric typewriters.

The creation of the position of professional assistant has considerably lightened the load of the executive secretary and has enabled the office to give better service in some areas.

B. Membership:
Membership of the Council is experiencing an increase during this year alone which is approximately equal to our total membership only ten years ago. The total members and subscribers should reach approximately 32,000 by May 1, 1960. The membership growth during each of the past four years has been, respectively, 17 per cent, 20 per cent, 22 per cent, and 31 per cent.

C. Financial Condition:
The financial condition of the Council is very satisfactory. The increase in membership and subscription receipts is almost directly proportional to the increase in number of members, or about 31 per cent. However, the increase in income from the sale of publications is about 70 per cent, indicating a growing interest among teachers in our publications. The sale of yearbooks accounts for about two-thirds of the income from our publication sales. At present time, it appears that the total income during the fiscal year from memberships, subscriptions, and sale of publications may reach $325,000.

D. Circulation of Journals:
The circulation of each of the three journals has increased steadily, in step with the growing membership. Below are the print orders for the issues that are now on the press: *Mathematics Teacher*, April 1960, 27,000 copies; Arithmetic Teacher April 1960, 14,500 copies; *Mathematics Student Journal*, March 1960, 75,000 copies.

E. New Publications:
The chief new publication for this year has been the 25th Yearbook, *Instruction in Arithmetic*. The number of copies printed was 10,000. The book is off the press and copies have been shipped to Buffalo for sale at the 38th Annual Meeting. The book has been advertised in the journals and in the program for the 38th Annual Meeting, with the statement that copies will be delivered after

the presentation of the book at the meeting. The editing of the book was made much easier by having an editorial assistant in the Washington office.

The pamphlet *Five Little Stories*, by W. W. Strader, came off the press recently. The number of copies printed was 6,400. Other pamphlets added to our list of publications are the following *Mathematics Tests Available in the United States*, 10,000 copies; *New Developments in Secondary School Mathematics*, 3,000 copies; *Mathematics for the Academically Talented Student in the Secondary School*, 9,000 copies.

On the press at the present time are the two bibliographies, *The High School Mathematics Library* and *The Elementary School Mathematics Library*. By a previous action of the Board of Directors, copies of these two bibliographies are to be mailed free to members of the Council, those members receiving *The Mathematics Teacher* to get the first bibliography and those who take THE ARITHMETIC TEACHER to receive the second.

Also on the press at the present time is the pamphlet *Notes on Vectors, Analytics, and Calculus for Twelfth-Year Mathematics*, by Abraham W. Glicksman, and the *Number Story*, by Herta T. and Arthur E. Freitag. In the files are two additional manuscripts which will be prepared for the press within the very near future. Their tentative titles are: *A Look at Modern Mathematics*, by Lyman C. Peck, and *What Is Number Theory?* by I. A. Barnett.

Publications reprinted: The following publications have been reprinted since the report of the Executive Secretary was made a year ago: 23rd Yearbook, 5,000 copies; 24th Yearbook, 10,000 copies; *How to Study Mathematics* (On the press), 10,000 copies; *How to Use Your Bulletin Board*, 5,000 copies; *Paper Folding for the Mathematics Class*, 7,000 copies; *How to Use Your Library in Mathematics*, 7,000 copies; *A Guide to the Use and Procurement of Teaching Aids for Mathematics*, 15,000 copies.

V. *Report of the 1960 Committee on Nominations and Elections.*

The following report was presented by Mrs. Ida Bernhard Puett, Chairman of the Committee.

In accordance with the Bylaws and regulations by the Board of Directors concerning election procedures, the 1960 election of officers and directors was conducted. The official count of the results was made by the Remington Rand Corporation. A report of the count was transmitted to the president of the Council, the chairman of the Committee on Nominations and Elections and to the executive secretary of the Council.

The following persons have been elected for a term of two years:

President: Phillip S. Jones, Ann Arbor, Michigan.

Vice-President for Senior High School: William H. Glenn, Pasadena, California.

Vice-President for Elementary School: Mrs. Clarence Ethel Hardgrove, DeKalb, Illinois.

The new directors serving a three-year term are: Robert E. K. Rourke, Kent, Connecticut; Irvin H. Brune, Cedar Falls, Iowa; J. Houston Banks, Nashville, Tennessee.

Appreciation is expressed to members of the 1960 Nominations and Elections Committee and to the staff in the Washington office.

(Secretary's Note: The election of Phillip S. Jones as President created a vacancy for one year in the Office of Vice-President for Colleges. The Board of Directors filled this vacancy by selecting Clifford Bell.)

VI. *Resolutions.*

Dr. Ben A. Sueltz presented the following resolution:

WHEREAS the National Defense Education Act of 1958 has been instrumental in assisting many schools to obtain valuable equipment for the teaching of mathematics, and

WHEREAS less than one-fifth of the money expended under the provisions of Title III has been used for the purchase of equipment for mathematics, and

WHEREAS typewriters with mathematical characters would be of great usefulness to the teacher in the preparation of tests, manuscripts, and other materials in mathematics,

BE IT RESOLVED that the members of the National Council of Teachers of Mathematics respectfully request the Commissioner of Education to seek a broader interpretation of the provisions of Title III to permit the purchase with funds made available through the Act of typewriters equipped with special mathematical characters.

Following the reading of this resolution, Dr. Sueltz moved its adoption. The motion was duly seconded and passed.

VII. *Report of the Resolutions Committee.*

The Resolutions Committee was composed of Miss Annie John Williams and Dr. Clifford Bell. Miss Williams gave the report of the Committee which follows:

We, the members of the National Council of Teachers of Mathematics, wish to express our sincere appreciation for the hospitality extended to us by the Association of Mathematics Teachers of New York State and the Buffalo Public Schools. We especially wish to thank Louis F. Scholl and his co-chairman, Theresa L. Podmele, and the members of their various committees for the efficient manner in which they are carrying out the careful plans made for the convention. This has done much toward making the meeting a very successful one.

Your city lives up to our expectations and many of us have enjoyed Niagara Falls and the other wonderful tours you have arranged for us.

We also wish to thank the press, the radio and television stations which have co-operated so generously.

After reading the report, Miss Williams moved its adoption. The motion was duly seconded and passed.

VIII. Dr. H. C. Christofferson, former president, moved that the President and the Board be extended a vote of thanks and complimented highly on the outstanding work that was accomplished during the past year. In addition, he moved that Dr. Ben A. Sueltz, retiring Editor of The Arithmetic Teacher, be given a vote of praise for the outstanding job he did in developing this fine Journal as its first editor. (Secretary's Note: Dr. Sueltz served two terms as Editor, which is the maximum time allowed under our Bylaws.)

The motion was duly seconded and passed.

IX. *Adjournment.*

The meeting was duly adjourned at 5:10 P.M.

Respectfully submitted,
Houston T. Karnes
Recording Secretary

The 1960–61 Budget

The Budget Committee for 1960 was composed of Dr. Bruce E. Meserve, Dr. H. Vernon Price, and Dr. Houston T. Karnes, Chairman.

The Committee met at the Washington Office, January 21–22, 1960. During this period, the Committee had the fine services of the Executive Secretary, M. H. Ahrendt, and the availability of all the important records in the Central Office.

On the basis of the work done during this meeting, a proposed budget was prepared and submitted, by mail, to the Board on March 4, 1960. At the Buffalo Meeting, the Budget was presented by the Chairman and discussed thoroughly by the Board. The Budget as adopted by the Board, which includes a few changes from the Budget as prepared by the Committee, is submitted below.

Receipts		
Memberships (26,000 @ $5.00)	$130,000	
Memberships (2,500 @ $1.50)	3,750	
M.S.J. Subscriptions (70,000 @ 30 cents)	21,000	
Memberships (for second journal: 5,000 @ $3.00)	15,000	
Subscriptions (both journals: 5,000 @ $7.00)	35,000	
Subscriptions (both journals: 5,000 @ $6.75)	33,750	
Advertising in Journals	10,000	
Interest on U. S. Treasury Bonds	375	
Interest on saving accounts	2,350	
Total		$251,225
Expenditures		
Washington Office		
Executive Secretary		
Salary	$ 11,275	
Professional Assistant		
Salary	7,825	
Editorial Assistant		
Salary	5,750	
Secretarial, editorial and clerical		
Salaries	55,000	
General		
General office expenses	25,000	
Travel	4,000	
Special benefits (Social Security, hospitalization and retirement)	8,500	
Total		$117,350

President		
Office expenses	$ 1,000	
Secretary	4,000	
Travel	2,000	
President's Fund	1,000	
TOTAL		$ 8,000
Vice-Presidents		
Office expenses	$ 800	
TOTAL		$ 800
The Mathematics Teacher		
Printing and mailing	$ 45,000	
Edit, Inc.	3,600	
Secretary	1,000	
Office expenses and equipment	800	
Travel	750	
TOTAL		$ 51,150
THE ARITHMETIC TEACHER		
Printing and mailing	$ 22,500	
Edit, Inc.	3,300	
Secretary	2,000	
Office expenses and equipment	1,000	
Travel	750	
TOTAL		$ 29,550
Mathematics Student Journal		
Printing and mailing	$ 15,000	
Office expenses	200	
Travel	750	
Experimental program	500	
TOTAL		$ 16,450
Other		
Committee expenses	$ 7,925	
Travel: committees	8,000	
Subcommittee work	5,000	
Travel: Vice-Presidents, Directors, Recording Secretary	5,000	
Convention program expenses	2,000	
TOTAL		$ 27,925
GRAND TOTAL		$251,225

Annual Financial Report

By M. H. Ahrendt

The report below gives a brief picture of the financial condition of the Council for the fiscal year June 1, 1959 to May 31, 1960. Readers who wish to compare this report with the similar report for the previous fiscal year will be able to make some interesting comparisons.

Both receipts and expenditures increased, in step with the rapidly growing membership. The fact that your officers were able to operate without increasing expenses in proportion to the increased receipts made it possible for a significant amount to be added to our cash resources, greatly strengthening the financial condition of the Council.

It is interesting to note that the per cent of increase in the sale of publications was more than double the per cent of increase in membership and subscription receipts, thus indicating a growing acceptance on the part of teachers of mathematics of our yearbooks and pamphlets.

RECEIPTS AND EXPENDITURES
OF THE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
FISCAL YEAR, JUNE 1, 1959–MAY 31, 1960

June 1, 1959 Total Cash Resources			\$ 99,579.36
RECEIPTS			
Memberships with subscriptions to *The Mathematics Teacher*	\$86,846.23		
Memberships with subscriptions to THE ARITHMETIC TEACHER	28,158.80		
Institutional subscriptions to *The Mathematics Teacher*	33,524.84		
Institutional subscriptions to THE ARITHMETIC TEACHER	30,759.45		
Subscriptions to *The Mathematics Student Journal*	20,088.46		
Sale of advertising space in *The Mathematics Teacher*	13,092.89		
Sale of advertising space in THE ARITHMETIC TEACHER	3,000.14		
Rental of membership list	1,829.24		
Interest on investments	1,467.77		
Net profit from conventions	3,452.37		
Miscellaneous	679.30		
Sale of publications			
Yearbooks	58,859.80		
Miscellaneous	32,770.01		
TOTAL RECEIPTS		\$314,529.30	
EXPENDITURES			
Washington office	\$92,426.40		
Purchase of office equipment	2,511.43		
President's office	5,414.44		
Vice-presidents' office expenses	1,228.84		
The Mathematics Teacher	44,107.33		
THE ARITHMETIC TEACHER	21,450.53		
The Mathematics Student Journal	11,366.79		
Committee work	2,540.50		
Travel by Board members	6,100.14		
Chicago Policy Conference	2,917.10		
Convention program expenses	968.30		
Contributions to White House and In-service Conferences	1,100.00		
Preparation and printing of yearbooks	29,848.33		
Preparation and printing of supplementary publications	17,323.35		
Storage and shipment of publications, miscellaneous	3,011.42		
Total expenditures		\$242,314.90	
Increase in Cash Resources			\$ 72,214.40
May 31, 1960—TOTAL CASH RESOURCES			\$171,793.76